MODELLING OF POST-CLOSURE BEHAVIOUR FOR MUNICIPAL SOLID WASTE LANDFILLS: SETTLEMENT AND SLOPE STABILITY

PhD Thesis

Author: Li Yu
Supervisors: Dr. Jesús Carrera
            Dr. Francisco Batlle

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HYDROGEOLOGY GROUP
TECHNICAL UNIVERSITY OF CATALONIA
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To my beloved family.

献给我挚爱的家人。
Abstract

Sanitary landfilling is the most economic method for the disposal of municipal solid waste (MSW). Once the waste is deposited, organic solids contained in the waste begin to biodegrade and continuously generate landfill gas. Hence, long-term settlement of MSW landfill exhibits both mechanical and biological components, so that MSW landfills behave like dynamic systems. Therefore, topics related to post-closure settlement and slope stability, which are of major concern to geotechnical engineers, do not follow conventional geotechnics laws. This study aims to investigate hydro-mechanical (HM) behaviour of MSW landfills and provide solutions to various aspects related to landfill design and operation.

After investigating the settlement mechanism of waste and clarifying the relationship between mechanical and biodegradation induced settlements, the effect of waste degradation on settlement was implicitly incorporated into a K-H rheological settlement model, which was then applied to several topics related to post-closure behaviours of MSW landfill:

- Temporal and spatial variations in unit weight of the MSW were estimated by considering the deformation of the waste matrix, which includes both short- and long-term mechanical effects, and mass loss caused by waste degradation. Typical parameter values were obtained for representative MSW landfills. The results reveal different variation trends of waste unit weight in the upper and lower portions.

- One-dimensional settlement for multi-layered MSW landfills was predicted analytically considering the interactions between the gas phase and the solid phase. The model reproduced both the time evolution of settlement and gas flux in horizontal landfill gas collection systems.

- Two-dimensional radial gas flow to a vertical gas extraction well in deformable MSW landfills was simulated analytically. The analytical solution shows that the economical optimum results from balancing imposed vacuum, well distance and cover properties.

- A hybrid method for quasi-three-dimensional slope stability analysis was proposed for dynamic evaluations of slope stability at a MSW landfill. Application of the model indicates that the method can serve as an engineering tool in preliminary estimates of safety factor and the position and extent of the potential slide mass.

- Numerical simulations of a long-duration pumping test with variable pumping rate at a MSW landfill were carried out to investigate the HM behaviour and determine the hydraulic parameters of waste. The hydraulic conductivity was found varying between $4.5 \times 10^{-7} - 5.5 \times 10^{-6}$ m/s with an anisotropic ratio of 10.

Research work carried out on a variety of topics enhanced the overall understanding of the post-closure behaviour of the MSW landfill. Model predictions compared well with field measurements in terms of unit weight, long-term settlement and gas extraction from wells, which proves the applicability of the proposed models in MSW landfills. The obtained waste parameters will help the engineers in the design and operation of MSW landfills.
Resumen

Los vertederos sanitarios constituyen el método más extendido y económico para el desecho de residuos sólidos urbanos (RSU). Tras el vertido, la componente orgánica comienza a degradarse y generar gases, por lo que el asiento a largo plazo de los RSU exhibe una componente mecánica y otra biológica, lo que da lugar un comportamiento dinámico. Por ello, todo lo relacionado con asientos y estabilidad a largo plazo, que son de interés geotécnico, no permite enfoques geotécnicos tradicionales. En esta tesis se ha investigado el comportamiento hidromecánico (HM) de los vertederos de RSU y se han desarrollado soluciones a varios aspectos relacionados con el diseño y operación de vertederos.

Tras investigar el mecanismo de asiento del residuo y aclarar la relación entre los asientos inducidos por biodegradación y por causas mecánicas, se optó por incorporar el efecto de la degradación de manera implícita en un modelo reológico de asiento K-H, que se aplicó a varios temas relacionados con el comportamiento de vertederos de RSU:

- Se estudiaron las variaciones espaciales y temporales del peso específico del RSU resultantes de la deformación mecánica a corto y largo plazo y de la pérdida de masa por degradación. Se obtuvieron valores representativos para diversos vertederos y se observó la diferencia de comportamiento entre la parte superior, donde disminuye el peso específico, y la inferior, donde tiende a aumentar.

- Se predijo analíticamente el asiento vertical de vertederos multicapa, considerando las interacciones entre las fases sólida y gas. El modelo reproduce tanto la evolución temporal de los asientos como el flujo de gas a sistemas horizontales de captación.

- Se resolvió analíticamente el problema de flujo radial bidimensional de gas hacia un pozo vertical de extracción. Esta solución muestra que el sistema óptimo resulta del equilibrio entre distancia entre pozos, vacío impuesto y propiedades de la cubierta.

- Se propuso un método híbrido para análisis dinámico cuasi-tridimensional de estabilidad de ladera en vertederos de RSU. La aplicación del método ha confirmado su interés práctico para la evaluación preliminar del factor de seguridad y la posición y extensión de posibles deslizamientos.

- Se realizaron simulaciones numéricas de un ensayo de bombeo de larga duración en un vertedero de RSU para investigar el comportamiento HM y sus parámetros hidráulicos. Se obtuvieron conductividades entre $4.5 \times 10^{-7}$ y $5.5 \times 10^{-6}$ m/s con un factor de anisotropía de 10.

El trabajo realizado ha permitido avanzar el conocimiento del comportamiento de los vertederos de RSU tras el cierre. Las predicciones numéricas reproducen bien las observaciones entérminos de peso específico, asiento a largo plazo, y producción de gas, lo que prueba la aplicabilidad de los modelos propuestos. Los parámetros obtenidos deberán ayudar al diseño y operación de futuros vertederos.
Resum
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CHAPTER 1

Introduction

1.1 Motivation and objectives

Municipal Solid Waste (MSW), more commonly known as trash or garbage, consists of everyday items we use and then throw away. The quantity of household waste generated ranged between 300 kg and 500 kg per inhabitant in most of the European Union (EU) Member States in 2008 (Data from European statistics 2010). The large majority of waste in the EU is landfilled, incinerated or recycled. In 2006, 60% of the MSW produced in Spain was deposited in landfills because it is believed to be more economical than other alternatives such as incineration and composting. At a sanitary landfill, the waste is placed on the ground and extended in thin layers (cells). These layers are then compacted to reduce their volume and covered periodically with a suitable material.

Once the waste is deposited, organic solids contained in the waste begin to degrade through bio-chemical reactions. As a consequence, MSW landfills suffer large long-term settlement which is associated with the volume reduction of solids. Simultaneously, landfill gas (LFG), which contains green-house effect gasses, such as methane and carbon dioxide, generates continuously through anaerobic biodegradation of the organic fraction of waste. The degradable characteristic of waste brings up several topics of interest for the design and operation of the MSW landfills, such as prediction of long-term settlement, dynamic evaluations of the slope stability of waste fills and gas emission control, etc.

Continuous degradation of solids distinguishes MSW from ordinary soil-like materials. Waste exhibits a strong time-dependent behaviour. Over the years numerous models have been developed to simulate the landfill settlement. Early models estimated the rate and magnitude of settlement relying on either classic soil mechanics or empirical methods. Rheological models and laboratory fitting curves have been used extensively for the long-
term settlement. These simple models have been successful to a limited extend, more in simulating measured settlement at specific sites than in predicting the general time behaviour of refuse settlement in relation to biological and physicochemical processes within a landfill (El-Fadel and Khoury 2000).

The more recent trend in MSW landfill modelling moves towards much more sophisticated models, which have a coupled framework for integrated analysis of hydro-bio-mechanical (HMB) behaviour of MSW landfill. These models track the behaviour of wastes as they degrade and compress in a more fundamental way. However, the inherent uncertainties associated with model parameters that can adequately describe the complex biochemical processes in landfills make the application of such models not easy in the real world. Most landfills are not sufficiently characterized to grant complete parameterizations for such complex models.

Previous studies conclude that there are three important mechanisms governing the deformation of waste in MSW landfills: (1) mechanical deformation due to surcharge loading; (2) time-dependent deformation due to creep of the waste skeleton; (3) time-dependent deformation due to biological degradation of waste. Because there is no rigorous model describing the relationship between the mass loss and the corresponding compressive strain, for simplicity, it is often supposed that the loss in solid volume totally converts into bulk volume reduction of the waste. Degradation induced deformation is therefore clearly distinguished from the time-dependent deformation due to creep and simply superimposed on the latter.

The procedure that assumes that the total settlement is the sum of mechanical induced-settlement and biological induced-settlement is questionable because from the mechanical point of view, mass loss does not induce settlement itself. The newly formed void volume resulting from solids degradation does not translate to subsidence directly. Instead, mass loss causes porosity of the waste to increase, and stiffness of solid matrix to decrease. That is, biodegradation of solid mass weakens the micro-structure of waste through conversion of the solid mass into biogas. The subsequent settlement is the result of collapse of the weakened waste skeleton under stress.

The subjective separation of the whole settlement process is not necessary, because mechanical compression and biodegradation-induced compression happen simultaneously during the whole progress of landfill settlement. Both are the result of the rearrangement of solid skeleton and time dependent. Thus it is reasonable to seek simplifications such as treating deformation due to biodegradation as the whole time dependent volume change (Hettiarachchi et al., 2007 & 2009), or applying a single rheological model to simulate the overall time-dependent deformation due to both creep and biodegradation (Durmusoglu et al., 2005; Yu et al., 2009 & 2010).
Based on the mechanisms of waste settlement, it is proposed in this thesis that the process of degradation and collapse be represented by a K-H rheological model (VOIGT form of the standard linear solid model). It reflects both the time-dependent and stress-dependent deformation, and requires limited parameters which are easy to determine. The response of settlement to the void enlargement may be locally erratic. However, a smoothed time-strain curve represented by the K-H rheological model is capable of describing the overall behaviour of settlement. This simplified method could serve as a phenomenological approach to evaluate the mechanical behaviour and mathematically describe the time-dependent behaviour of MSW landfills.

MSW is an interacting multiphase medium (gas-liquid-solid-heat) with each phase exhibiting significant temporal and spatial variations. The waste settlement is accompanied with LFG generation and dissipation, and migration of leachate within a landfill. However most of the existing settlement models focus only on the compressibility of the solid phase of waste. Models available to evaluate gas generation and transportation in the MSW landfill neglect the large compressibility of the landfill. Building a multi-phase model which considers the interaction between mechanical behaviour and gas generation and dissipation within the MSW landfill is therefore the primary objective of the current research.

Coupling between the time-dependent settlement based on the K-H rheological model and generation and dissipation of the LFG was further applied in this thesis to various aspects related to the design and operation of MSW landfills. This thesis presents analytical solutions to the temporal and spatial variations of MSW unit weight, to the predictions of long-term settlement, and to the gas flow around a vertical landfill gas extraction well. These solutions were validated through comparisons with field measurements. Besides these, a three-dimensional hybrid method for estimating the safety factor of slope considering continuous settlement of waste was developed for MSW landfills. Coupled hydro-mechanical responses of MSW landfill during a long-duration pumping test were also analysed. Research work carried out on such a variety of topics aims to enhance the overall understanding of the post-closure behaviour of the MSW landfill, thus help the optimal design and operation of the MSW landfills.

1.2 Thesis outline

This thesis is an integration of five independent while interconnected chapters, each one dealing with a various aspect related to the MSW landfill.

In chapter 2, a model for assessing the temporal and spatial variations of MSW unit weight was developed, which considers the variations of unit weight caused by deformation
of the waste matrix and degradation of the organic portion. The methodology used for the settlement modelling due to solids degradation serves as the basis of modelling the mechanical behaviour of MSW in the following chapters.

In chapter 3, a coupled model for prediction of long-term settlement and gas flow in one-dimensional MSW landfills was presented. This chapter put emphasis on the analytical solutions to the prediction of long-term settlement in multi-layered MSW landfills. Comparisons between model predictions and field measurements at several sites were given.

In chapter 4, analytical solutions for two-dimensional radial transient gas flow to a vertical gas extraction well in deformable MSW landfills were presented. Parametric studies were performed to benefit the optimal design of the final cover and extraction wells.

In chapter 5, a hybrid method for quasi-three-dimensional slope stability analysis based on the finite element stress analysis was applied in a case study. Influences of time elapsed after landfill closure, variations of leachate level and variations of unit weight of waste on safety factor of slope stability were also investigated.

Chapter 6 presented the results of a long-duration pumping test with variable pumping rate performed in a MSW landfill. Numerical simulations were conducted in order to get a better understanding of the coupled hydro-mechanical behaviour of waste during pumping.

Publications during the doctoral stage are listed below. The first three papers contribute respectively to chapters 2, 3 & 4. Papers 4 and 5 below deal with analytical methods and numerical tools which are the basis of this Ph.D. thesis. Papers 6 and 7, which have not been published yet, contain the material in chapters 5 and 6.


CHAPTER 2

Variations of waste unit weight during mechanical and degradation processes at landfills

Abstract

This manuscript developed a model for assessing the time and space variations in unit weight of traditional municipal solid waste (MSW) landfills. The model considers the variations of waste unit weight caused by deformation of the waste matrix and degradation of the organic portion. Deformation of waste matrix includes both short-term effects, resulting from mechanical strain during the filling period, and long-term effects, resulting from superposition of waste skeleton creep and waste degradation. Mass loss, caused by waste degradation, not only affects the stress level within the waste column, but also induces large and long-term deformation. Degradation-induced deformation is caused by the local collapse of the solid matrix weakened by mass loss. Considering that the correlation between mass loss and waste deformation is locally erratic and hard to define, a smooth time-strain curve (represented by Kelvin viscoelastic model) was used to describe approximately the overall long-term deformation. The analytical formulation for unit weight was obtained in the Laplace transform domain and can be used to simulate spatial and temporal variations of waste unit weight. Unit weight profiles obtained at four MSW landfills using the proposed model agree well with

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measurements from *in situ* large-scale unit weight tests. Evolution of unit weight profiles indicates that there is no monotonous varying trend for unit weight along the whole depth of the landfill. Density first increases and then decreases to a stable value in the lower portion, whereas the opposite occurs in the upper portion. Whether unit weight increases or decreases depends on the competition between matrix deformation and degradation processes.

**Key words:** municipal solid waste landfill; unit weight; compressive strain; degradation; viscoelastic model; analytical formulation

### 2.1 Introduction

The unit weight of municipal solid waste (MSW) is the weight (kN) per unit volume (m³) of a bulk waste mixture including waste and the cover soil (some moisture present but waste not saturated). MSW unit weight is a basic and important parameter for many landfill analyses, including landfill capacity evaluation, slope/subgrade stability calculations, determination of hydraulic conductivity and integrity calculations of liner, cover system, drainage system, landfill gas system and geosynthetic interface within the base liner system, etc. Surprisingly, few detailed studies have been conducted on MSW unit weight (Dixon and Jones, 2005). As a result, MSW unit weight remains a major source of uncertainty in landfill performance analyses (Fassett et al., 1994). One of the common difficulties in assessing MSW unit weight summarized by Fassett et al. (1994) is the assessment of the variations in unit weight with time and depth.

Several empirical unit weight profiles have been reported in the literature in order to guide the estimation of unit weight in a typical MSW landfill. Unit weight, for old waste samples obtained up to a depth of 30 m by means of a 0.3 m diameter bucket auger ranges between 5 kN m⁻³ and 18 kN m⁻³ (Oweis and Khera, 1986). A linear fit to the above data versus depth yields a range from 7 kN m⁻³ at the surface to 17 kN m⁻³ at a depth of 30 m, but a correlation coefficient of only 0.35 (Burlingame et al., 2007). Based on shear wave velocity measurements, a nonlinear unit weight profile was suggested by Kavazanjian (1999) with unit weights varying from 12-16 kN m⁻³ near the ground surface to 14-17 kN m⁻³ at a depth of 60 m. Zekkos et al. (2006) performed a thorough review of reliable field data and concluded that each individual landfill displays a characteristic MSW unit weight profile. They suggested an empirical hyperbolic equation to represent the relation between unit weight and depth, and recommended three representative unit weight profiles corresponding to landfills with low, typical, high near-surface unit weight. The resulting empirical unit weight profiles give
geotechnical engineers a simple method for practically estimating the spatial variations of waste unit weight within landfills, but do not yield its temporal variation.

Unit weight profile for a waste column can be determined from the variations of volumetric strain with depth. Most work on landfill settlement prediction emphasizes the calculation of the total average strain over the entire waste thickness (Oweis, 2006, Park and Lee, 2002; Marques et al., 2003), but such calculations do not allow the calculation of strains at different depths. Bleiker et al. (1995) simulated density variations for each refuse layer by considering the deformation of each layer in response to the weight of overlying waste. They used the Gibson and Lo model (1961) to simulate the time dependence of the stress-strain relationship. However, they did not consider mass loss due to waste degradation. Durmusoglu et al. (2005) derived a coupled numerical model to consider solid-gas-liquid interactions within MSW landfills. They used the Maxwell viscoelastic model to simulate the compressive strain of the whole system, although it yields an infinite final deformation. Their work illustrated the evolution of density profiles for a hypothetical case: density decreases in the early years as a result of mass loss, and increases in the later years due to landfill settlement. Hettiarachchi et al. (2009) developed a computer program to keep track of the mass variations of each MSW layer for bioreactor landfills at variable liquid and gas pressure conditions, and thus numerically predicted the variations of unit weight within the landfill. In their calculation, the landfill settlement includes two parts: mechanical deformation due to the overburden stress, simulated by the compression and swelling ratios obtained from laboratory compression tests, and biodegradation-induced deformation. They assume that the void volume created by waste degradation will totally transform into skeleton deformation, but do not consider the influence of effective stress on the process of degradation-induced deformation.

Over the past ten years, sophisticated models were developed with a coupled framework for integrated analysis of hydro-bio-mechanical (HMB) behaviour of MSW landfill. These models overcome the limitations of treating biodegradation related landfill settlement as a simple time-dependent process by tracking the behaviour of wastes as they compress and degrade in a more fundamental way. In a recent solid waste modelling challenge, different approaches were used to predict the outcomes of a well constrained laboratory experiment on the settlement and biodegradation. Among them, numerical software by McDougall (2008), Lobo et al. (2008) and White (2008) are capable of modelling the hydraulic, biodegradation and mechanical behaviour of landfilled waste. Different techniques were adopted in these models regarding the mechanical consequences of degradation. McDougall (2008) considered elastic, plastic, creep and degradation-induced strains in the mechanical module. Void phase volume was linked to solid phase volume through a decomposition-induced void (DIV) change parameter. Lobo et al. (2008) only included primary settlement and biodegradation-induced settlement in the settlement model. The latter was directly obtained by multiplying a factor by the mass loss. Degradation-induced settlement in White (2008) was linked to mass
loss through the dry density which was empirically calculated from stress state. Beaven (2008) provided a valuable summary of the state of art in landfill process modelling as well as current uncertainties and research needs.

In this paper, an alternative model to calculate the temporal variations of compressive strain with respect to depth within MSW landfills was proposed. This, in turn, allows the prediction of temporal and spatial variations of waste unit weight. The proposed model accounts for the mechanical strain caused by weights from subsequent lifts during filling, and time-dependent strain due to both creep and degradation of waste during and after filling. Solutions of strain and unit weight at any time and depth within the MSW landfill were obtained analytically in the Laplace transform domain. Furthermore, unit weight profiles obtained by the proposed model were compared with in situ measurements at four MSW landfills, so as to test the applicability of the proposed model in the practice.

2.2 Model development

Waste unit weight undergoes spatial and temporal variations after the waste is deposited in the landfill through the mechanical compaction, ravelling, physicochemical change, biochemical decay and interaction among these mechanisms (El-Fadel and Khoury, 2000). It is normally believed that there are two predominant, simultaneous mechanisms that control the variation of waste unit weight in MSW landfills: waste matrix deformation and mass loss due to waste degradation. Actual variations of unit weight depend on the competition between increase due to matrix deformation and decrease due to degradation.

2.2.1 Degradation-induced mass loss

Degradation of MSW is governed by several chemical and microorganism mediated reactions through which solid forms of biomass are solubilised and converted into gases such as methane and carbon dioxide. Hydrolysis is the key mechanism that starts biochemical conversion of a solid substrate. From a recent international modelling challenge to model and predict the performance of a well-constrained laboratory experiment on the settlement and biodegradation of MSW, one of the main differences identified relates to hydrolysis. Three different algorithms were used by six different groups: first-order decay, a modified enzymatic hydrolysis function and Monod kinetics (Beaven, 2008). Four among six groups chose a first-order decay algorithm.
A widely used approach for first-order decay algorithm is based on the superposition of first-order kinetics terms, each representing a fraction of the waste (Findikakis and Leckie, 1979; Hettiarachchi et al., 2009)

\[
\Omega(t') = \rho_0 \sum_{i=1}^{n} \beta_i \left(1 - e^{-\lambda_i t'}\right)
\]

(2-1)

where \(t'\) (year) is the time elapsed since waste deposition; \(\Omega(t')\) (kg m\(^{-3}\)) is the mass loss per unit volume at time \(t'\) since deposition; \(\rho_0\) (kg m\(^{-3}\)) is as-placed bulk waste density which will be assumed homogeneous within the waste column; the number of the waste group based on degradability is denoted by subscript \(i\), \(i=1, \ldots, n\); \(\beta_i\) (%) and \(\lambda_i\) (year\(^{-1}\)) are mass fraction and first order kinetic constant for the \(i\)th waste group, respectively.

Suppose that a MSW landfill is constructed with a constant filling rate. The time needed for completion is \(t_f\). Waste at the lower part of the landfill is deposited earlier, thus, waste age and decay rate at the lower part are different from the waste deposited in the upper layers. To account for the different decay rates of waste at different depths within the same landfill, time \(t'\) in Eq. (2-1) is defined as (Arigala et al., 1995)

\[
t' = t + \frac{z}{H} t_f
\]

(2-2)

where \(t\) (year) is the time elapsed since landfill closure; \(z\) (m) is depth; and \(H\) (m) is the final thickness of the landfill. After closure, the mass loss at depth \(z\) and time \(t\) can be estimated by substituting \(t'\) by Equation (2-2) in Equation (2-1) as

\[
\Omega(z,t) = \rho_0 \sum_{i=1}^{n} \beta_i \left(1 - e^{-\lambda_i z/t_f}\right)
\]

(2-3)

### 2.2.2 Deformation of waste matrix

Previous studies conclude that there are three important mechanisms governing the deformation of waste in MSW landfills.

1. Instantaneous response to load, which refers to the mechanical deformation due to surcharge loading; for example, the weight of each subsequent lift during filling. This type of deformation is complete at or shortly after completion of filling (Wall and Zeiss., 1995). Although the mechanisms behind this type of deformation in MSW are not the same as those
of soils, Terzaghi theory can be considered empirically applicable because it provides reasonable estimates of deformation and typical ranges for its parameters are well established (Wall and Zeiss, 1995).

(2) Time-dependent deformation due to creep of the waste skeleton, which is associated with slippage or the reorientation of the particles (Bjarngard and Edgers, 1990). An extension of Terzaghi’s theory, which assumes that the compressive strain is linear with respect to the logarithmic time, was first suggested by Sowers (1973) to predict such a kind of deformation.

(3) Time-dependent deformation due to biological degradation of waste, which refers to the large and long-term volume reduction induced by the mass loss due to degradation of waste. To evaluate this portion of deformation, Bjarngard and Edgers (1990) used a similar formulation as for calculating creep deformation, but adopting a higher secondary compression index. As there is no rigorous model describing the relationship between mass loss and the corresponding compressive strain, for simplicity, it is often supposed that the loss in solid volume totally converts into bulk volume reduction of the waste (Marques et al., 2003; Oweis 2006; Park and Lee, 2002). Degradation induced deformation is therefore clearly distinguished from the time-dependent deformation due to creep and simply superimposed on the latter.

For degradable materials such as MSW, traditional approaches combining the latter two processes are believed to be unhelpful in developing a sound understanding of the long-term settlement behaviour of waste (Powrie et al., 2009). However, creep and biodegradation-induced volume reduction are interacted and proceed simultaneously. Both are the result of the rearrangement of solid skeleton and time dependent. It is not easy to distinguish the settlement consequences between creep and waste degradation. Thus it is reasonable to seek simplifications such as treating deformation due to biodegradation as the whole time-dependent volume change (Hettiarachchi et al., 2007 & 2009), or applying a single rheological model to simulate the overall time-dependent deformation due to both creep and biodegradation (Durmusoglu et al., 2005; Yu et al., 2009 & 2010).

The experiment data provided by Ivanova et al. (2008) clearly indicate the importance of mechanical creep in the settlement mechanism (Powrie et al., 2009). The models by Lobo et al. (2008) and White (2008), which treat the biodegradation-induced settlement as the whole secondary settlement, underestimate the settlement behaviour because they do not incorporate the creep as a mechanism for secondary settlement. It is concluded that there appears to be a clear imperative for models that predict settlement to include a mechanical component in their algorithm (Powrie et al., 2009).

In the following, the total compressive strain in the landfill is assumed to consist of two parts: strain $\varepsilon_1$, referring to the instantaneous compressive strain due to loading, and strain $\varepsilon_2$, corresponding to the time-dependent strain due to both creep and degradation.
2.2.3 Mechanical deformation during filling

Most compression test results on real and artificial MSW samples in the literature show an almost linear relationship between the instant waste volume change and the logarithm of the effective vertical stress, which is comparable to the instant compressive behaviour of soils. It is for this reason that the concept of compression index extended from classic soil mechanics has so far been used by researchers and practitioners to represent MSW compressibility and then used to analyse primary landfill settlement (Zhang et al., 2010).

One of the limitations for using classic soil mechanics to estimate mechanical compression for MSW is that household refuse usually comprises a great many compressible component (e.g. plastic packaging, cans and boxes). By separating the volumetric change due to compressible waste particles from that due to inter-voids (defined as voids within waste particles), it was found that volumetric changes for both show linear relationship with vertical stresses (logarithm), the similar trend as the total volume change (Zhang et al., 2010). Therefore, a linear relationship between strain and logarithmic stress is still valid for modelling the transient compression behaviour of MSW in one-dimensional problem.

Instantaneous compressive strain (taken hereinafter as negative) caused by surcharge is described here by Sowers (1973, Morris and Woods (1990), Oweis and Khera (1986), Wall and Zeiss (1995), etc.:

\[
\varepsilon_1(z) = -C'_c \log\left(\frac{\sigma'(z)}{\sigma_p'}\right)
\]

(2-4)

where \( C'_c \) is modified compression index; \( \sigma'(z) \) (kPa) is the final effective stress at depth \( z \), and \( \sigma_p' \) (kPa) is the initial effective stress which is assumed \( \sigma_p' = 1 \) kPa. Only relatively dry MSW landfill with moisture content lower than field capacity is considered in this work. Therefore, the effect of pore water pressure can be ignored and total stress \( \sigma \) can be used instead of effective stress in Equation (2-4).

Wall and Zeiss (1995) remarked that both instantaneous mechanical deformation and time-dependent deformation, including creep and biological degradation, occur simultaneously. The magnitude of the former is greater and masks the effects of the latter during the initial period. Thus, the parameter \( C'_c \) derived from field measurements would include impacts of waste degradation to some degree. In fact, \( C'_c \) would increase with time as mass loss progresses (Oweis, 2006). The value of \( C'_c \) to be used in Equation (2-4) during this...
period does not include degradation, which is considered as \( \varepsilon_{2-0} \), value of the cumulative time-dependent strain occurred during filling, and explained in the following chapter.

Suppose that stress within the MSW landfill is caused only by the weight of waste without considering surface surcharge and final top cover. Stress at any depth \( z \) at the end of filling can be determined by integrating waste mass and taking into account of the mass loss due to degradation:

\[
\sigma(z) = \int_0^z [\rho_0 - \Omega(z)]g dz = \rho_0 gz - \Delta \sigma(z)
\]  

(2-5)

where \( \Delta \sigma \) is the reduction in stress due to mass loss. The latter results from Equation (2-3) for \( t=0 \), which leads to

\[
\Delta \sigma(z) = \sum_i \beta_i \rho_0 gz - \sum_i \rho_0 \beta_i gH \frac{1}{\lambda_i t_f} \left(1 - e^{-\lambda_i t_f H}ight)
\]

(2-6)

Compressive strain at the end of filling is thus obtained by substituting Equations (2-6) and (2-5) into Equation (2-4)

\[
\varepsilon_1(z) = -C_e \log \left[ \frac{1}{\rho_p} \left( \sum_i \beta_i \rho_0 gz + \sum_i \rho_0 \beta_i gH \frac{1}{\lambda_i t_f} \left(1 - e^{-\lambda_i t_f H}ight) \right) \right] / \sigma_p
\]

(2-7)

### 2.2.4 Time-dependent deformation

Continuous degradation of solid matter distinguishes MSW from ordinary soil-like materials. Waste exhibits an obvious time-dependent behaviour. There is no rigorous model describing the relationship between mass loss and the corresponding settlement of the landfill. McDougall and Pyrah (2004) proposed a new parameter \( \Lambda \), degradation-induced void (DIV) change parameter, to describe the relationship between decomposed solid phase, \( dV_s \) and the induced change in void volume, \( dV_v \) as \( dV_v = \Lambda \cdot dV_s \). The authors also indicated that the value of \( \Lambda \) is not constant but changes with the progress of degradation.

For most settlement models considering degradation, they calculate mechanical-induced settlement and degradation-induced settlement separately. Normally, there is a basic assumption for degradation-induced settlement: it is directly proportional to the amount of solids degradable. For simplicity, it was assumed that the newly formed void volume due to degradation (i.e. volume fraction of degradable content) totally transforms into landfill
settlement (Hettiarachchi et al., 2007 & 2009; Oweis, 2006). This is the particular case for \( \Lambda=0 \) in the phase relation model by McDougall and Pyrah (2004).

From the mechanical point of view, mass loss does not induce settlement itself. The newly formed void volume resulting from solids degradation does not transform into subsidence directly. Instead, mass loss causes porosity of the waste to increase, and stiffness of solid matrix to decrease. That is, biodegrading of solid mass weakens the micro-structure of waste through conversion of the solid mass into biogas. The subsequent settlement is a result of collapse of the weakened waste skeleton under stress (Yu et al., 2009). Settlement is controlled by the interaction of soil skeleton strength and stress state (McDougall and Pyrah, 2004).

The response of settlement due to collapse of the weakened micro-structure waste skeleton may be locally erratic (McDougall and Pyrah, 2004); however, a smoothed time-strain curve is capable of describing the overall behaviour of settlement at macroscopic level. In this paper, the time-dependent deformation of MSW is represented by a Kelvin model as illustrated in Figure 2-1. It consists of two parallel basic elements, namely the Hookean spring, characterized by its modulus \( E \), and a dashpot, characterized by its viscosity \( \eta \). The two parallel joined elements simulate a system with time variable stiffness. The elastic modulus of the spring is called residual elastic modulus of the solid matrix. It determines the final and constant strain of the system.

![Figure 2-1: Schematic representation of Kelvin viscoelastic model.](image)

The basic working principle of Kelvin viscoelastic model is that a time-dependent settlement is obtained by continuously decreasing the matrix stiffness. The decreasing rate is controlled by the viscosity of the dashpot. Blight (2005) provided a case study showing the decrease of the waste stiffness with time. The simulated time-settlement curve using Kelvin model is comparable to the typical curve characterized for MSW landfills. The comparison between model prediction and field measurements at several MSW landfills proves that the model can well reproduce the time evolution of settlement (Yu et al., 2009 & 2010).

The implicit form of the Kelvin viscoelastic model is given by
\[ \sigma = -E \varepsilon - \eta \dot{\varepsilon} \]  

(2-8)

where \( \sigma \) is vertical stress; \( \varepsilon \) is compressive strain of the solid skeleton; \( E \) (kPa) is residual elastic modulus of the solid matrix; and \( \eta \) (kPa\cdot s) is viscosity of solid skeleton. The advantages of using Kelvin model in modelling compressive behaviour of waste include: 1) the strain described by Kelvin model reaches a finite long term value, which can be chosen to be the ultimate averaged strain observed in MSW landfills; 2) there is a close relationship between decay rate of waste and viscosity of the waste matrix; namely the value of \( E/\eta \) can be related to \( \lambda \); 3) the parameters required in the Kelvin model are quite limited and easy to determine through \textit{in situ} tests or fitting from field data (Yu et al., 2010).

The stiffness of waste matrix is not constant, but depends upon the mean stress (Fasset et al., 1994; Dixon and Jones, 2005). Therefore, the residual elastic modulus in Equation (2-8) is assumed to vary with depth. It can be expressed in a general form as:

\[ E = f(E_0, z) \]  

(2-9)

where \( E_0 \) (kPa) is residual elastic modulus of the solid matrix at the landfill surface. Data on values of \( E \) for MSW, including Young’s modulus (Fassett et al., 1994) and drained constrained modulus (Dixon and Jones, 2005), as a function of vertical stress are summarized in Figure 2-2. It is also implicit in Figure 2-2 that Young’s modulus of waste increases with the initial density of the waste (Fassett et al., 1994).
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Figure 2-2: Increase of waste stiffness with stress level.
2.2.5 Time-dependent strain accumulated during filling

During filling of the landfill, stress within the landfill keeps varying with time because of the waste depositing process as well as mass loss due to degradation. Stress at any time $\bar{t}$ and depth $z$ can be approximated by assuming a linear growth of stress during filling for the whole landfill.

$$\sigma(z,\bar{t}) = \int_0^z \left[ 1 - \sum_i \beta_i \left( 1 - e^{-\lambda_i \bar{t}/H} \right) \right] \frac{\bar{t}}{t_f} \rho_0 g dz$$ \hspace{1cm} (2-10)$$

where $\bar{t} (0 < \bar{t} < t_f)$ is time elapsed since the placement of waste during filling period. The time-dependent strain occurred during filling can be obtained in the Laplace transform domain by substituting the transform of Equation (2-10) into Equation (2-8)

$$\tilde{\varepsilon}_{2-0}(z) = \left\{ \frac{1}{f(E_0, z) + s \eta} \right\} \left[ 1 - \sum_i \beta_i \left( 1 - \sum_i \frac{\beta_i H}{\lambda_i t_f} \left( 1 - \exp\left(-\lambda_i t_f z / H \right) \right) \right] \frac{\rho_0 g}{t_f s^2}$$ \hspace{1cm} (2-11)$$

where $\tilde{\varepsilon} = \int_0^\infty \varepsilon(t)e^{-st} dt$ is the Laplace transform of $\varepsilon$ and $s$ is Laplace frequency. $\tilde{\varepsilon}_{2-0}(z)$ is the Laplace transform of time-dependent strain at depth $z$ during filling. Its value at the end of filling represents the accumulated time-dependent strain during filling and is denoted as $\tilde{\varepsilon}_{2-0}(z)$. This is obtained by inverting Equation (2-11) and setting time equal to $t_f$. Crump’s method (Crump, 1976) is used for numerical Laplace inversion.

2.2.6 Time-dependent strain after closure

Stress within the landfill keeps varying with time due to the mass loss during degradation. After closure, total stress within the landfill at any time $t$ and depth $z$ is determined by substituting Equation (2-3) into Equation (2-5) as

$$\sigma(z,t) = \int_0^z \left[ 1 - \sum_i \beta_i \left( 1 - e^{-\lambda_i (t + t_f z / H)} \right) \right] \rho_0 g dz$$ \hspace{1cm} (2-12)$$

By introducing Equation (2-12) into Equation (2-8), the time-dependent strain can be expressed explicitly in the Laplace transform domain as
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\[
\tilde{\varepsilon}_{2-1}(z) = -\frac{\rho_0 g}{f(E_0, z) + s\eta} \left( z \left( 1 - \sum_{i} \beta_i \right) \frac{1 - e^{-\lambda_i t_f t_H}}{s} + \sum_{i} \beta_i \left( 1 - e^{-\lambda_i t_f t_H} \right) \left( s + \lambda_i \right) \lambda_i t_f t_H \right) \tag{2-13}
\]

where \( \tilde{\varepsilon}_{2-1}(z) \) is the Laplace transform of time-dependent strain occurred after closure. Its value at any time \( t \) and depth \( z \) is obtained by inverting Equation (2-13) and denoted as \( \varepsilon_{2-1}(z,t) \).

### 2.2.7 Unit weight profiles

Bulk unit weight of waste within a MSW landfill is controlled by two simultaneous effects: increase of unit weight due to the deformation of the solid skeleton and decrease of the unit weight as a result of degradation. The deformation of the waste, as a function of time and depth, is the sum of \( \varepsilon_1(z) \), \( \varepsilon_{2-0}(z) \) and \( \varepsilon_{2-1}(z,t) \) which have been given in Equations (2-7), (2-11) & (2-13), respectively. \( \varepsilon_1(z) \) is the mechanical strain at the end of filling; \( \varepsilon_{2-0}(z) \) refers to the accumulated time-dependent strain during filling; \( \varepsilon_{2-1}(z,t) \) is the time-dependent strain occurred after closure. Mass loss during degradation of waste is described by Equation (2-3). Therefore, the time variation of unit weight after closure can be computed as:

\[
\gamma(z,t) = \frac{1 - \sum_{i} \beta_i \left( 1 - e^{-\lambda_i t_f t_H} \right)}{1 + \varepsilon_1(z) + \varepsilon_{2-0}(z) + \varepsilon_{2-1}(z,t)} \rho_0 g \tag{2-14}
\]

Similarly, surface settlement can be obtained by numerically integrating the total strain over the whole landfill thickness.

\[
S(t) = \int_0^H \left( \varepsilon_1(z) + \varepsilon_{2-0}(z) + \varepsilon_{2-1}(z,t) \right) dz \tag{2-15}
\]

### 2.3 MSW unit weight data

*In situ* large-scale methods are judged to be the most reliable approaches for evaluating the unit weight of MSW (Zekkos et al., 2006). This section presents available unit weight data at four MSW landfills obtained by *in situ* large-scale tests.
2.3.1 In situ unit weight measurements at Coll cardús landfill

To characterize the geotechnical properties of the waste material at Coll cardús landfill, in situ large-scale unit weight tests were performed in 2007 as part of this study. The test results are provided in this paper for the first time. Coll Cardús sanitary landfill is an active municipal and industrial solid waste disposal centre in the region of Catalonia, located in a natural valley in east-northern Spain. It accepts more than 600,000 tones per year of both municipal and industrial solid waste since the mid 1980s. The area of the landfill is approximately 200,000 m$^2$. The depth of the waste varies with the topography of the original valley from 40 to 70 m. Two large-diameter boreholes for unit weight test were drilled using bucket auger with a net diameter of 760 mm. The volume of the waste material retrieved from the borehole was evaluated by the large-scale 'volume replacement' method recommended by Zekkos et al. (2006) through backfilling with pre-calibrated gravel. Bulk volume of waste material of approximate 0.5 m$^3$ was retrieved repeatedly from the borehole at an interval of about 5 m to a maximum depth of 30 m below the ground surface.

In situ unit weight measurements were made at eight locations during the drilling of the two boreholes: six in the first borehole and only two in the second because of the appearance of the leachate at a depth of 18 m in the second borehole. When drilling below the leachate level, waste material crept into the borehole from the surroundings. As a result, an extremely large amount of waste was retrieved. Evidence from the waste material retrieved in the first borehole shows that the waste age is no more than five years. The fraction of waste samples larger than 15 mm was weighted and characterized visually. Figure 2-3 displays the composition of waste recovered from the first borehole, including soil and stone (about 30% by weight), textile (about 30%), plastic (about 30%) and paper (about 10%). It is clear that the composition varies significantly and exhibits obvious layered character, especially the content of soil and stone. This reflects the layered filling procedure at the site. Moisture content was measured separately for waste samples larger and smaller than 15 mm, yielding ranges of 9.5-41.7 % and 18.5-27.9 %, respectively. The organic content of waste samples smaller than 15 mm varies between 3.13 % and 31.4%. The measured unit weight varies from 15 kN m$^{-3}$ to 21 kN m$^{-3}$ with an average value of 18.5 kN m$^{-3}$, and exhibits great heterogeneity from layer to layer (Figure 2-4). These unit weight values have not been corrected of overbore because the waste is quite stiff in the unsaturated portion. These values fall in the upper region of the database collected by Zekkos et al. (2006).
2.3.2 Other available in situ unit weight data

Unit weight measurements for three U.S. landfills, 'Older', Azusa and OII, obtained through *in situ* large-diameter borehole test are briefly summarized herein. Detailed information of test procedure and unit weight data are reported by Matasović and Kavazanjian (1998), Zornberg et al. (1999), Oweis and Khera (1998) and Zekkos et al. (2006).

The OII landfill (Matasović and Kavazanjian, 1998), located near Los Angeles, accepted residential, commercial, and industrial solid wastes from the middle of the last century until 1984. The landfill site was approximately 60 m deep and filled with solid waste over a 40-year period. Six *in situ* unit weight measurements were made in each of the three 840-mm-diameter bucket auger borings to depths of up to 45 m. The volume of the MSW removed from a 2 to 3 m length of borehole was evaluated by backfilling with pre-calibrated gravel. However, the volume of the 2 to 3 m interval of the boring from which the waste was removed was finally calculated by assuming a constant 16\% overbore because liquids were observed seeping into the boring at a relatively rapid speed at several locations. The results of the unit weight were adjusted and varied between 12 and 21 kN m\(^{-3}\). Measured moisture contents were 15-42\%. Azusa landfill (Zornberg et al., 1999) started at the same time as OII landfill, also near Los Angeles, and was closed in 1995. The height of the landfill reaches about 80 m. The waste has been placed in layers and separated by daily cover soils. A total of 19 *in situ* unit weight measurements were performed during the drilling of two of the gas extraction wells between 8 m and 50 m below the landfill surface. Each measurement corresponds to the average unit weight of the waste material retrieved in boring segments.
approximately 3 m long. The results show that unit weight of waste ranges from 10 to 15 kN m\(^{-3}\). The results indicated a roughly constant unit weight profile (~15 kN m\(^{-3}\)), most likely due to the relatively large amount of cover-soil that was placed in the landfill (Zekkos et al., 2006). Boring logs showed that the waste material is mainly solid, household waste, green waste, and inert waste. Gravimetric moisture contents were 8-50% with an average of 28%. Another unidentified landfill in New Jersey, denoted as 'Older' landfill (Oweis and Khera,
1998), suggested MSW unit weights near the surface of about 7 kN m$^{-3}$, with a more pronounced increase in the unit weight with depth than at OII or Azusa (Zekkos et al., 2006).

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Unit weight measurements for these landfills, together with field-fitted hyperbolas based on the proposed unit weight model by Zekkos et al. (2006), are provided in Figure 2-4. These three MSW landfills are considered to be representatives of low, typical and high near-surface unit weight, which is dependent upon waste composition and operation procedures.
2.4 Application of the unit weight model to existing MSW landfills

Application of the proposed model in reproducing unit weight profiles for MSW landfills was examined at the above four MSW landfills. For a typical MSW landfill, only two waste groups, that is $i=2$, were considered in the simulation: one is degradable group with decay rate of $\lambda$, and the other is non-degradable group with decay rate equal to zero. In the absence of site-specific data, a default value of 0.05 year$^{-1}$, which is equivalent to a half life of 13.6 years was suggested for $\lambda$ (Oweis, 2006). As-placed unit weight of waste was taken approximately as the unit weight measurement at the landfill surface. Time information was determined approximately from construction records.

The linear relationship between residual elastic modulus and depth, $E = f(E_0, z)$, is extensively used in geotechnical problems (Mayne and Poulos, 1999; Rowe, 1982):

$$E = E_0 + k_e \cdot z$$

where $E_0$ (kPa) is the value of residual elastic modulus at the surface the MSW landfill ($z=0$); $k_e$ (kPa m$^{-1}$) is the rate of increase of modulus with depth (unit of $E$ per unit depth); and $z$ (m) is depth. The three straight lines in Figure 2-2 were used to represent variations of residual elastic modulus with depth in the simulation for MSW landfills with low, typical and high near-surface unit weight, respectively.

Other parameters were adjusted to visually fit field data within a reasonable range. Both Coll Cardús landfill and OII landfill had high near-surface unit weight and hence, mechanical parameters were taken to be the same for both landfills. As-placed unit weight was assumed homogeneous within the waste column. Unit weight profiles computed using the proposed model are shown in Figure 2-4 as solid lines for each landfill. All the parameter values used in the simulation are listed in Table 2-1.

For the case of Coll Cardús landfill, spatial variations of strain 2 and 50 years after closure are presented in Figure 2-5, together with three components of strain, namely, mechanical strain at the end of filling, $\varepsilon_1$, accumulated time-dependent strain during filling, $\varepsilon_{2,0}$, and post-closure time-dependent strain, $\varepsilon_{2,1}$. Two years after closure, $\varepsilon_1$ and $\varepsilon_{2,0}$ constitute the major portion of the total strain, and post-closure strain, $\varepsilon_{2,1}$, has just started (Figure 2-5(a)). However, post-closure strain will evolve with time and become increasingly significant in the long term (Figure 2-5(b)).
### Table 2-1: Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
<td><strong>Values</strong></td>
</tr>
<tr>
<td>Height of the MSW landfill (m)</td>
<td>'Older' Azusa OII Coll Cardús</td>
</tr>
<tr>
<td>Fraction of degradable mass, $\beta$ (%)</td>
<td>30</td>
</tr>
<tr>
<td>Decay constant of the waste, $\lambda$ (year$^{-1}$)</td>
<td>0.051</td>
</tr>
<tr>
<td>Time needed to construct the landfill, $t_f$ (years)</td>
<td>30 30 35 10</td>
</tr>
<tr>
<td>Time after closure, $t$ (years)</td>
<td>30 2 5 2</td>
</tr>
<tr>
<td>As-placed density, $\rho_0$ (kg m$^{-3}$)</td>
<td>780 1150 1520 1750</td>
</tr>
<tr>
<td>Modified compression index, $C_c'$</td>
<td>0.07 0.06 0.035 0.035</td>
</tr>
<tr>
<td>Residual elastic modulus at landfill surface, $E_0$ (kPa)</td>
<td>5 15 20 20</td>
</tr>
<tr>
<td>Linearly increasing rate of stiffness with depth, $k_E$ (kPa m$^{-1}$)</td>
<td>30 50 80 80</td>
</tr>
<tr>
<td>Viscosity of solid skeleton, $\eta$ (kPa•s)</td>
<td>$4 \times 10^{11}$ $6 \times 10^{11}$ $8 \times 10^{11}$ $8 \times 10^{11}$</td>
</tr>
</tbody>
</table>

![Figure 2-5: Variations of strain with depth at Coll Cardus landfill ($\varepsilon_1$ is the mechanical strain during filling, $\varepsilon_{2,0}$ is the time-dependent strain accumulated during filling and $\varepsilon_{2,1}$ is the time-dependent strain after closure): (a) 2 years after closure; (b) 50 years after closure.](image)
Evolution of settlement at Coll Cardús landfill can be easily obtained through Equation (2-15). As indicated in Figure 2-6, the settlement of the landfill becomes stabilized some 50 years after closure with an ultimate total average strain of about -30% (total average strain is defined as the integration of strain from bottom to top divided by the whole depth of the landfill). The total average mechanical strain and time-dependent strain happened during the filling period of 10 years, $\varepsilon_1$ and $\varepsilon_{2-0}$, are -9% and -6.7%, respectively (negative means compressive strain). By the end of filling period, the landfill has already undergone a total average strain of -15.7%. The time variation of the total average strain after closure is shown in the thick solid line.

![Figure 2-6: Evolution of total average strain with time at Coll Cardús landfill.](image)

**2.5 Temporal and spatial variations of waste unit weight**

Unit weight of MSW increases with matrix deformation, but decreases with mass loss during degradation. Actual increases or decreases depend on time and space. The overall trend results from the competition between these two contrary effects. Figure 2-7 and Figure 2-8 illustrate the simulated spatial and temporal variations of unit weight at a MSW landfill with a typical near surface unit weight (using the parameters of Azusa landfill in Table 2-1).

Temporal variations of unit weight profile are presented in Figure 2-7 as well as the temporal variations of strain profile. As observed in the right graph, compressive strain increases with depth as a result of the increase in stress level. It also increases with time due to degradation induced deformation. The shape of the unit weight profile varies with time and
becomes increasingly curved. Although the compressive strain increases with time along the whole depth of the landfill, time variation trends of unit weight vary with depth. In the upper part, unit weight decreases with time because mass loss overrides matrix deformation under a relatively lower stress level.

The decreasing of unit weight with time obtained in the uppermost layers was also observed by McDougall (2009). In that paper, the author gave out a quantitative impact of mass loss on phase composition through a series of laboratory tests on partly soluble sand. It was found that dissolution leads to a looser or more open soil skeleton and the void ratio is at all times greater than its initial value. Furthermore, there is a remarkable consistency in the four test data with different vertical stresses of 12, 50, 112 and 237 kPa.

![Figure 2-7: Temporal and spatial variation of unit weight and compressive strain in the MSW landfill](image)

**Figure 2-7: Temporal and spatial variation of unit weight and compressive strain in the MSW landfill (Parameters are the same as those for Azusa landfill in Table 2-1).**

Temporal variations of unit weight can be examined more carefully at five different depths in Figure 2-8. Unit weight at different depth exhibits different evolution characteristics. Furthermore, there is no monotonous variation trend even at one certain depth. In the lower portion of the waste column, unit weight keeps increasing with time in the first 20 years. Then it is followed by a small decrease and afterwards becomes stabilized. Variation trends in the upper portion are opposite. Unit weight first decreases with time immediately after closure.
Then, it increases a little bit. At depth of about 17 m, the post-closure unit weight remains virtually constant with time. Further measurements of unit weight versus depth over time are required to provide evidences for the model prediction. This result is quite consistent to opinions by Zekkos et al. (2006) that rational mechanisms by which MSW unit weight increases, decreases, and does not change with degradation can all be postulated.

![Temporal variation of density at different depth in the MSW landfill](image)

**Figure 2-8**: Temporal variation of density at different depth in the MSW landfill (Parameters are the same as those for Azusa landfill in Table 2-1).

### 2.6 Model discussion

Over the years numerous models have been developed to simulate landfill settlement. The early models estimate the rate and magnitude of settlement relying on either classic soil mechanics or empirical methods. Rheological models and laboratory fitting curves have been used extensively for the long-term settlement. These simple models have been successful to a limited extend, more in simulating measured settlement at specific sites than in predicting the general time behaviour of refuse settlement in relation to biological and physicochemical processes within a landfill (El-Fadel and Khoury, 2000).

The latest trend in MSW landfill modelling moves towards much more sophisticated models, which have a coupled framework for integrated analysis of hydro-bio-mechanical (HMB) behaviour of MSW landfill. These models track the behaviour of wastes as they degrade and compress in a more fundamental way. However, the inherent uncertainties associated with estimating model parameters that can adequately describe the complex biochemical processes in landfills make the application of such models not easy in the real world. Most landfills are not always sufficiently characterized to grant complete parameterizations for such complex models.
The simplified rheological model and analytical solution proposed in this paper use the phenomenological approach to evaluate the mechanical behaviour and mathematically describe the time-dependent behaviour of MSW landfills. It considers the variation of stress state within waste column due to degradation, as well as the stress-dependent creep process. The intrinsic simplicity of such models determines that such models can not provide as much information as those coupled HBM models which intend to build a complete and fundamental processes modelling for MSW landfill. Comparing to the great computing effort necessary for coupled HBM models, such simplified models are suitable for preliminary and scoping calculations and are consistent with engineering practice.

2.7 Summary and conclusions

Appropriate prediction of waste unit weight profiles is a first step for any other engineering analysis of MSW landfill performance. This paper provided a model to simulate the temporal and spatial evolution of unit weight through calculating time variations of strain with respect to depth within MSW landfill. The total strain includes mechanical strain caused by the weight of subsequent lifts during filling, and time-dependent strain due to both creep and degradation of waste during and post filling. The effect of waste degradation on strain is time-dependent and implicitly incorporated into the rheological model. Time variations of the waste unit weight at any depth within a MSW landfill depend on the competition between two contrary effects: decrease due to degradation and increase due to mechanical deformation of the solid matrix.

Solutions of strain and unit weight within the MSW landfill are expressed analytically in the Laplace transform domain. Typical parameter values were obtained for three representative MSW landfills with low, typical and high near-surface unit weight. The good agreement between model simulations and in situ data using conventional parameters suggests that the proposed model is applicable in practice. Although compressive strain within the waste column keeps increasing with depth and time, there is no single variation trend of waste unit weight along the whole depth of the landfill. Unit weight decreases with time in the upper portion of the landfill because degradation overrides mechanical deformation. The opposite occurs in the lower portion of the waste.

The model does not include water flow within the MSW landfill. Therefore, it is restricted to relatively dry and fully drained landfill. More field measurements of unit weight versus depth over time are required to calibrate the proposed model.
CHAPTER 3

Evaluation of post-closure settlement in MSW Landfills

Abstract

Prediction of long-term settlement and control of gas pollution are two principle concerns during the management of municipal solid waste (MSW) landfills. The behaviour of settlement and gas flow in MSW landfills is complicated due to the combined effect of mechanical deformation of the solid skeleton and continuous biodegradation of the waste. A one-dimensional settlement and gas flow model was presented in this paper, which is capable of predicting time evolution of settlement as well as temporal and spatial distribution of gas pressure within multi-layered landfills under a variety of operating scenarios. The analytical solution to the novel model was evaluated with numerical simulation and field measurements. The resulting efficiency and accuracy highlight the capability of the proposed model to reproduce the settlement behaviour and vertical gas flow in MSW landfills. The influences of operating conditions and waste properties on settlement and gas pressure were examined for typical MSW landfills.

Key words: municipal solid waste landfill; settlement; gas flow; Laplace transform; analytical formulation

3.1 Introduction

Sanitary landfills are recognized to be an economic method for the disposal of municipal solid waste (MSW). Prediction of long-term settlement and control of gas pollution to the environment are two principle concerns during the management of municipal solid waste (MSW) landfills.

MSW landfills suffer large long-term settlement that is associated with volume reduction caused by biodegradation of organic solids, and also by creep of the MSW skeleton (Sower 1973; Park and Lee 2002). The impact of excess settlement on liner system and final cover, landfill gas (LFG)/leachate extraction system as well as post-closure development of MSW landfill is significant. Therefore, accurate estimation of settlement becomes critical to the successful operation and further maintenance of the MSW landfills. Early models used to predict long-term settlement for MSW landfills are either adjusted from soil mechanics (Sower 1973) or empirical functions based on best fit approximation of field measurements (Yen and Scanlon 1975; Edil et al., 1990; Ling et al., 1998). These models are used extensively in the practice for their simplicity. Abundant experiences have been accumulated in the determination of the model parameters.

For the prediction of gas flow within MSW landfills, many models have been proposed by assuming that waste is porous medium and gas velocity is governed by Darcy’s law. In addition to the models that simulate gas flux around vertical gas extraction well (Arigala et al., 1995; Nastev et al., 2001; Chen et al., 2003), some other models were developed to predict gas flux in horizontal gas extraction systems. Townsend et al. (2005) gave an analytical solution for steady-state one-dimensional gas flow to assist in the assessment and design of horizontal LFG collection systems. Findikakis and Leckie (1979) presented a one-dimensional multi-component gas flow model to simulate the vertical distribution of gas pressure and gas composition. Young (1989) modelled advective gas flow in MSW sites when a series of horizontal extraction wells are present.

The settlement and gas flow models mentioned above focus only on one phase neglecting the fact that the MSW landfill is an interacting multiphase medium (solid, gas, and liquid) with each phase exhibiting significant temporal and spatial variations (El-Fadel and Khoury 2000). Recently some work was reported on landfill settlement incorporating the fluid flow in MSW landfills. Durmusoglu et al. (2005) gave a one-dimensional multi-phase numerical model considering the interactions among solid, gas and liquid phases. In this model, Maxwell viscoelastic model, which describes an infinite compression of solid skeleton with time, was used as mechanical constitutive law. The three-phase settlement model by Hettiarachchi et al. (2007 & 2009) used the laboratory compression curve as the constitutive
law for stress-induced settlement. The coupling effect between mechanical compression and fluid flow was built with the assumption that the void volume formed during biodegradation transforms totally into the landfill settlement. Liu et al. (2006) proposed a solid-gas coupled model based on the linear unsaturated consolidation theory. In the model, mechanical compression of the waste was assumed to vary linearly with gas pressure. Yu et al. (2009) developed a model to predict two-dimensional radial transient gas flow to a vertical gas extraction well in deformable MSW landfills using K-H rheological model. The analytical solutions given in Laplace transform domain facilitate the optimum design of LFG systems. McDougall (2007) gave a conceptual framework for coupled analysis in relation to hydraulic, biodegradation and mechanical processes. In the framework, an unsaturated flow model, a two-stage anaerobic digestion model, and a mechanical model consisting of load, creep, and biodegradation-induced settlement are incorporated. Bente et al. (2007) are carrying out a numerical simulation considering mechanical deformation and reactive transport processes within MSW landfills.

A typical landfill is deposited in layers and usually takes up to decades to be filled. It is very common that at the same spot, the new cell is constructed many years after the previous cell was finished. During the time span before construction of the new layer, the old waste has already undergone some degree of degradation. Permeability, waste density and mechanical properties of the old waste are much different from those in the newly constructed cell. Therefore, it is much more reasonable to assume that the properties of waste over the life of the landfill vary from layer to layer. Quite few existing models simulate the settlement and gas pressure in layer-deposited MSW landfill. After back analysis of parameters using Gibson and Lo model in four sites, Edil et al. (1990) pointed that old refuse has a lower secondary compressibility compared to refuse recently surcharged. Bleiker et al (1995) developed a settlement model to predict the compression of each refuse layer in response to the varying weight of overlying refuse in deep landfills.

Most of the existing multiphase models rely on numerical method (FEM/FDM). This feature makes the reproduction and application of these models inconvenient especially for engineers without much experience of numerical techniques. This paper presents an analytical solution based on multi-phase model to predict both settlement and gas pressure for single-layered as well as multi-layered MSW landfill under various operating scenarios. The analytical solution considers time-dependent compression of solid skeleton, as well as generation and dissipation of LFG. Analytical method serves as a valuable tool because it offers quick insight into the physics of the coupled system and can provide preliminary and scoping calculations. Moreover, the analytical solution given in this paper offers a helpful benchmark for validation of the solutions employing numerical techniques.
3.2 Model Development

Some basic assumptions are made to develop the governing equation:

1) Solid particles are assumed incompressible due to its much higher stiffness than solid skeleton for MSW;

2) LFG is normally considered to behave as an ideal gas under the pressures and temperatures commonly encountered in MSW landfill, and Darcy's law can be applied to the gas flow process (Hettiarachchi et al., 2009; Young 1989; Townsend et al., 2005; Findikakis and Leckie, 1979);

3) According to the study of Young (1989), the temperature within the MSW landfill tends to be fairly uniform (usually around 35-40°C). Therefore, the effects of temperature variation is neglected and isothermal condition was assumed;

4) Gas diffusion is disregarded based on the study of Townsend et al. (2005) that the maximum ratio of diffusive flux to advective flux is 1.1×10^-3 for landfill gas.

3.2.1 Mass balance equation

One-dimensional gas flow in a single-layered MSW landfill, as illustrated in Figure 3-1 was considered. The single-layered MSW landfill was assumed to be constructed instantaneously and at the same time. Mass balance equation for the gas phase is given by

\[
\frac{\partial (\theta_a \rho_a)}{\partial t} = -\frac{\partial (J_a)}{\partial z} + \alpha
\]

where \( \theta_a \) is volumetric gas content expressed as \( \theta_a = \phi S_g \), \( \phi \) is the porosity and \( S_g \) is degree of gas saturation defined as the volumetric fraction of voids occupied by gas phase; \( \rho_a \) is the density of gas (kg/m^3); \( J_a \) is gas flux in z direction (kg/m^2/s); and \( \alpha \) is source term of gas production rate due to waste biodegradation (kg/m^3/s).

One dimensional gas flux in z direction is written as

\[
J_a = -\frac{k_a}{\rho_a g} \left( \frac{\partial \rho_a}{\partial z} - \rho_a g \right)
\]
where \( k_a \) is gas conductivity in \( z \) direction (m/s) and is assumed to be constant along the depth of the waste column which can be estimated from pumping test or air-injection test; \( \bar{u}_a \) is absolute gas pressure (kPa) and expressed as \( \bar{u}_a = u_a + u_{atm} \), \( u_{atm} \) is atmospheric pressure (101.3 kPa), \( u_a \) is excess gas pressure (kPa); \( g \) is gravitational acceleration (9.807 m/s\(^2\)).

**Figure 3-1: One-dimensional gas flow in a single-layered landfill.**

Biodegradation of solid waste and gas generation in MSW landfills are governed by a series of chemical and biological reactions through which solid organic particles are solubilised and converted to methane and carbon dioxide. The most widely used approach for quantitatively describing gas production rate in MSW landfills is based on the first-order kinetic equation (Durmusoglu et al., 2005; Hettiarachchi et al., 2007 & 2009; Liu et al., 2006; Arigala et al., 1995; Findikakis and Leckie 1979; etc.)

\[
\alpha = G_T \lambda e^{-\lambda t} \tag{3-3}
\]

where \( G_T \) is gas production potential per unit volume of waste (kg/m\(^3\)); \( \lambda \) is reaction-rate constant of the waste (s\(^{-1}\)); \( t \) is time elapsed since waste deposition (s).

LFG is considered to behave as an ideal gas in the environment of MSW landfills. Therefore

\[
\rho_a = \frac{\bar{u}_a M}{RT} \tag{3-4}
\]

where \( R \) is universal gas constant equal to 0.008314 kJ/mol/K; \( T \) is absolute temperature of gas and assumed to be 310 K; \( M \) is average molecular weight of gas and taken to be 0.03 kg/mol.
Considering that porosity may change because of both compression of the solid matrix and enlargement of pore volume due to solid degradation, while using porosity for $\theta_a$, variations in mass storage for a dry MSW landfill ($S_g = 1$) will be given by

$$\frac{\partial (\theta_a \rho_a)}{\partial t} = \rho_a \frac{\partial \phi}{\partial t} + \phi \frac{\partial \rho_a}{\partial t} = \rho_a \left( \frac{\partial \varepsilon_a}{\partial t} + \frac{Y \alpha}{\rho_s} + \phi \frac{\partial \rho_a}{\partial t} \right)$$  \hspace{1cm} (3-5)$$

where $\varepsilon_a$ is volumetric strain of solid matrix; $\rho_s$ is the density of solid phase ($\text{kg/m}^3$); $Y$ is gas yield coefficient of refuse (mass of solid phase degraded/mass of gas phase generated and it is assumed to be 1.). The term, $Y \alpha / \rho_s$, represents the rate of volume enlargement due to degradation of the organic matter.

By using Equations (3-2)-(3-5), and neglecting gas flux due to gravitational gradient, Equation (3-1) can be written as

$$\rho_a \frac{\partial \varepsilon_a}{\partial t} + \phi \rho_a \frac{\partial \bar{u}_a}{\partial t} = k_a \frac{\partial^2 \bar{u}_a}{\partial z^2} + G \lambda e^{-\lambda t} \left[ 1 - \frac{Y \rho_a}{\rho_s} \right]$$  \hspace{1cm} (3-6)$$

Excess gas pressure in MSW landfill normally is a few kPa. Its variation can be neglected compared with $u_{atm}$ (101.3 kPa). Therefore, there exist $\bar{u}_a = \bar{u}_{a0}$ and $\rho_a = \rho_{a0}$, where $\bar{u}_{a0} = u_{atm} + u_{a0}$, $u_{a0}$ is initial excess gas pressure (kPa), $\rho_{a0}$ is initial gas density ($\text{kg/m}^3$). Considering that porosity of waste does not vary much under the combined effect of compression and mass loss, we assume a first-order approximation of porosity, that is $\phi = \phi_0$, where $\phi_0$ is the initial porosity. Rearranging Equation (3-6) gives

$$\rho_{a0} \frac{\partial \varepsilon_a}{\partial t} + \phi_0 \rho_{a0} \frac{\partial \bar{u}_a}{\partial t} = k_a \frac{\partial^2 \bar{u}_a}{\partial z^2} + G \lambda e^{-\lambda t} \left[ 1 - \frac{Y \rho_{a0}}{\rho_s} \right]$$  \hspace{1cm} (3-7)$$

### 3.2.2 Mechanical deformation

Continuous degradation of solid matter weakens the micro-structure of waste through conversion of the solid mass into biogas. The durable settlement that happened in MSW landfills is the result of collapse of the weakened micro-structure waste skeleton. There is no rigorous model describing the relationship between mass loss and the corresponding settlement of the landfill. The response of settlement due to collapse of the weakened micro-structure waste skeleton may be locally erratic, however, a smoothed time-strain curve is
capable of describing the overall behaviour of settlement at macroscopic level. In this paper, the settlement process in MSW landfills is represented by a viscoelastic K-H rheological model (VOIGT form of the standard linear solid model as illustrated in Figure 3-2), which reflects both the stress-dependent and the time-dependent deformations, and requires limited parameters easy to be determined. In K-H rheological model, the Hookean spring is used to simulate the primary settlement and the Kelvin element is used to simulate the secondary settlement.

![Figure 3-2: Schematic representation of K-H viscoelastic model.](image)

According to unsaturated soil mechanics of Fredlund and Rahardjo (1993), the compressibility of solid is controlled by two stress invariants: net normal stress, $\sigma-u_a$, and matric suction, $\psi$, where $\sigma$ is the total stress, $u_a$ is excess gas pressure. When there is no external load applied on the landfill, the total stress within the refuse is gravitational stress caused by self-weight. For the typical range of moisture content in MSW landfills, matric suction is negligible compared to net normal stress based on the retention curve measured by Kazimoglu et al. (2005). Thus, the volumetric strain that happened in the waste at depth of $z$ can be expressed as

$$\varepsilon_v = m_v^s \left( \rho_0 g z - u_a \right) \quad (3-8)$$

where $m_v^s$ is compression coefficient of solid skeleton (kPa$^{-1}$); $\rho_0$ is bulk density of refuse (kg/m$^3$).

The expression for volumetric strain can be split between that $\varepsilon_1$, corresponding to the Hookean spring of stiffness $E_0$, and $\varepsilon_2$, corresponding to the Kelvin element. There satisfy

$$\varepsilon_1 = \sigma'/E_0 \quad (3-9a)$$
\[ \sigma' = E\varepsilon_z + \eta\dot{\varepsilon}_z \]  

(3-9b)

where \( \sigma' \) is effective stress equal to \( \rho_0gz - u_a \) (kPa); \( E_0 \) is elastic modulus of waste for primary settlement (kPa); \( E \) is residual elastic modulus of the solid matrix for secondary settlement (kPa); \( \eta \) is viscosity of solid skeleton (kPa·s). The value of \( E_0 \) can be obtained from the primary settlement happened in the landfill, namely, in the first three months. The magnitude of \( E \) reflects the final settlement happened in the landfill which could be back analysed through best fitting the settlement evolution. The observed response of settlement in MSW landfill displays a very close relationship with the progress of biodegradation. Settlement is controlled by the interaction of soil skeleton strength and stress state (McDougall and Pyrah 2004). Here it is assumed that the stiffness of the solid matrix decreases at the same rate with the degradation of solid mass, there is \( E/(\lambda \eta) = 1 \). Therefore, the parameter \( \eta \) can be determined by the residual modulus and the degradation rate constant of waste.

The explicit expression of \( m_k^a \) can be expressed by employing Laplace transform. Transforming Equations (3-9a) and (3-9b), and adding the corresponding transformed strain, lead to

\[ \tilde{\varepsilon}_v = \tilde{m}_k^a \tilde{\sigma'} = -\left( \frac{1}{E_0} + \frac{1}{E + s\eta} \right) \left( \frac{\rho_0gz}{s} - \tilde{u}_a \right) \]  

(3-10)

where \( \tilde{\varepsilon}_v = \int_{-\infty}^{t} \varepsilon_v(t) e^{-st} \, dt \) is Laplace transform of the volumetric strain at depth \( z \), \( s \) is Laplace operator; \( \tilde{m}_k^a, \tilde{\sigma'} \) and \( \tilde{u}_a \) are Laplace transforms of corresponding variables as defined above. Here the increase of pore volume is assumed to be positive; thus, the coefficient \( \tilde{m}_k^a \) is negative.

Incorporating K-H rheological constitutive law of solid matrix (Equation (3-10)) into the gas mass balance equation (Equation (3-7)) in the Laplace transform domain gives

\[ \frac{\partial^2 \tilde{u}_a}{\partial z^2} - \frac{s}{C_v^a} \tilde{u}_a + \frac{C_v^i}{C_v^a} \tilde{u}_a^z + \frac{C_d}{C_v^{au}} \frac{1}{s + \lambda} + \frac{C_{uv}}{C_v^a} u_{ao} = 0 \]  

(3-11)
where \( C_v = -\frac{RTk_v}{(\bar{m}_k \bar{u}_{a0} - \phi_0)Mg} \); \( C_v^a = \frac{\rho_0 \bar{m}_k \bar{u}_{a0}}{\bar{m}_k \bar{u}_{a0} - \phi_0} \); \( C_a = -\frac{RT - \frac{Y \bar{u}_{a0}}{\rho_0}}{\bar{m}_k \bar{u}_{a0} - \phi_0} \).

Equation (3-11) will be used to find the excess gas pressure within the MSW landfill. The term \( \left(C_v^a / C_v\right)z \) in Equation (3-11) is the coupling term caused by the compression of solid matrix under gravitational stress. The term \( \left(C_d / C_v^a\right)/(s + \lambda) \) describes the effect of biodegradation of solid mass and the term \( C_{u0} \bar{u}_{a0} / C_v^a \) is generated due to the initial excess gas pressure. Equation (3-11) is able to demonstrate intrinsic relationship among a great number of parameters involved in the problem, and makes the prediction of effects due to variation of some key parameters straightforwardly.

The total compression of landfill in the Laplace transform domain for one-dimensional case can be expressed as

\[
\tilde{S} = \int_0^H \tilde{\varepsilon}_v(z) dz
\]

where \( \tilde{S} \) is the total compression of a waste column with height of \( H \); \( \tilde{\varepsilon}_v \) is Laplace transform of volumetric strain at depth \( z \) and is determined by Equation (3-10).

### 3.3 Solution Formulation

The proposed model developed above is firstly applied into a single-layered MSW landfill. Analytical solutions are obtained for three different operating scenarios. Furthermore, the analytical formulation will be extended to a multi-layered landfill system.

#### 3.3.1 Solutions for single-layered landfill

Solution for excess gas pressure within the MSW landfill requires specifying boundary conditions. The following three operating scenarios are commonly encountered in MSW landfills.
**Case 1:** Free-venting at top surface, and fixed gas flux is specified at the bottom. This is to simulate an influx of gas from a possible saturated layer at the base (Young 1989). This solution is useful for the dry part above leachate table in the landfill.

\[
\tilde{u}_a = 0 \text{ at } z=0 \quad (3-13a)
\]

\[
-\frac{k_a}{g} \frac{\partial \tilde{u}_a}{\partial z} = \tilde{J}_b \text{ at } z=H \quad (3-13b)
\]

where \(\tilde{J}_b\) is Laplace transform of the gas influx rate, \(J_b\) (kg/m²/s), at the base.

**Case 2:** Fixed gas pressure is specified at the top surface to simulate vacuum pressure applied beneath the impermeable geomembrane cap, and fixed gas pressure at the bottom base to simulate gas extraction through the leachate collection system (LCS) by applying the vacuum pressure to the LCS (Townsend et al., 2005).

\[
\tilde{u}_a = \bar{u}_i \text{ at } z=0 \quad (3-14a)
\]

\[
\tilde{u}_a = \tilde{u}_b \text{ at } z=H \quad (3-14b)
\]

where \(\bar{u}_i\) and \(\tilde{u}_b\) are Laplace transform of the vacuum pressure applied at the top and bottom, respectively.

**Case 3:** A thin layer with thickness \(d_i\) (m) and gas permeability \(k_i\) (m/s) is added at the top surface to simulate the low-permeable cover. Gas flow is predominantly vertical (Young 1989; Arigala et al., 1995) when the height of landfill \(H \gg d_i\) and thus the boundary condition at landfill cover is

\[
\frac{\partial \tilde{u}_a}{\partial z} = \frac{k_i}{d_i k_a} \tilde{u}_a \text{ at } z=0 \quad (3-15a)
\]

At the bottom of the landfill, it is assumed to be impermeable

\[
-\frac{k_a}{g} \frac{\partial \tilde{u}_a}{\partial z} = 0 \text{ at } z=H \quad (3-15b)
\]

The initial condition for excess gas pressure throughout the landfill for all three cases is defined as
At the time of closure; the initial excess gas pressure, \( u_{a0} \), is assumed to be zero in the following analysis. The solutions of excessive gas pressure, total compression of the waste column and gas flux at the top for one-layered landfill are obtained in the Laplace transform domain and given in Appendix 3-A. Crump’s method (Crump 1976) is used in this paper to perform the numerical inversion of Laplace transform in order to get the solutions in the real-time domain.

### 3.3.2 Solutions for multi-layered landfill

The multi-layered MSW landfill considered in this section is illustrated in Figure 3-3. It consists of \( n \) homogeneous and perfectly bonded layers. Any layer \( j ( j = 1, 2, \cdots, n ) \), occupying the region \( Z_{j-1} \leq z \leq Z_j \) with finite thickness \( H_j = Z_j - Z_{j-1} \), has its respective gas conductivity \( k_{aj} \), reaction-rate constant \( \lambda_j \), bulk density \( \rho_{0j} \), gas production potential \( G_{rj} \), and mechanical parameters \( E_{0j}, E_j, \eta_j \), etc.

![Multi-layered landfill diagram](Figure 3-3: Multi-layered landfill.)
Equation (3-2) and Equation (3-11) can be rewritten as a matrix ordinary differential equation

$$\frac{d\tilde{X}}{dz} = \tilde{A} \cdot \tilde{X} + \tilde{B} \tag{3-17}$$

where $\tilde{X} = \begin{bmatrix} \tilde{u}_a \\ \tilde{J}_a \end{bmatrix}$, $\tilde{A} = \begin{pmatrix} 0 & -g/k_a \\ -\frac{s}{C_v} & 0 \end{pmatrix}$ and $\tilde{B} = \begin{pmatrix} C_v k_a z + \frac{C_d k_a}{g} & 0 \\ \frac{C_v}{g} & \frac{1}{g} + \frac{k_a}{g} C_a u_{a0} \end{pmatrix}$.

The solutions of the matrix ordinary differential Equation (3-17) in the Laplace transform domain at depth $z$ within the $j$th layer ($Z_{j-1} < z < Z_j$) can be expressed as (Chen, 2003 & 2004; Yu and Chen, 2008)

$$\tilde{X}(z) = T_j(z - Z_{j-1}) \tilde{X}(Z_{j-1}) + S_j(z - Z_{j-1}) \tag{3-18}$$

where $\tilde{X}(Z_{j-1})$ is the vector of excess gas pressure and gas flux at the top surface of the $j$th layer in the Laplace transform domain; $\tilde{X}(z)$ is the vector of excess gas pressure and gas flux at depth of $z$ in the Laplace transform domain; $T_j$ and $S_j$ are transferring matrices of the $j$th layer (see Appendix 3-B).

The continuity conditions for any interface between the $j$th layer and $(j+1)$th layer is expressed as

$$\tilde{X}(Z_j^-) = \tilde{X}(Z_j^+) \tag{3-19}$$

By applying the forward transfer matrix method to the sequence of finite layers from the top surface of the multiple-layered landfill ($z=0$) downwards and by using the solution of the vector $\tilde{X}(0)$ given in Appendix 3-B, the vector solution, $\tilde{X}(z)$ at any depth $z$ within the $j$th layer ($Z_{j-1} < z < Z_{j+1}$) can be expressed as

$$\tilde{X}(z) = TT(z) \tilde{X}(0) + SS(z) \tag{3-20}$$

where $\tilde{X}(0)$ is the vector of excess gas pressure and gas flux at top surface of the multi-layered landfill in the Laplace transform domain. $TT$ and $SS$ take the forms:
\[ TT(z) = T_{j_1} \left(z - Z_{j_1}\right) T_{j_2} \left(H_{j_2}\right) \cdots T_{j_n} \left(H_{j_n}\right) T_{j_1} \left(H_{j_1}\right) \quad \text{for} \quad (Z_{j-1} < z < Z_{j+1}) \]  
(3-21a)

\[ SS(z) = \sum_{j=1}^{n} T_{j} \left(z - Z_{j}\right) T_{j} \left(H_{j}\right) \cdots T_{j} \left(H_{j}\right) S_{j} \left(H_{j}\right) \quad \text{for} \quad (Z_{j-1} < z < Z_{j+1}) \]  
(3-21b)

The expressions of transferring matrices \( T_j \) and \( S_j \), and the solution of the vector \( \tilde{X}(0) \) under three operating scenarios (Equations (3-13)-(3-16)) are presented in Appendix 3-B.

### 3.4 Model Testing

In this section, the correctness of the analytical solution is verified by comparison with results from the numerical method. The applicability of the solution for predicting long-term settlement and gas flux is verified through comparison with field measurements at two MSW landfills.

#### 3.4.1 Verification of the analytical solution

Solutions of settlement and excess gas pressure are compared with numerical results obtained with CODE_BRIGHT. CODE_BRIGHT is a well-established finite element code developed for calculating displacements, liquid pressure, gas pressure, temperature and salt content for boundary value problems in saturated or unsaturated soil (Olivella et al., 1996). Yu et al. (2011) investigated the validity of CODE_BRIGHT by comparing with results from TOUGH2 through three examples of thermo-hydro-gas transport modelling for a geological repository in Boom Clay.

In this study, modifications were implemented into CODE_BRIGHT to incorporate gas generation and porosity change due to degradation of solids. By “turning off” three sub-modulus of liquid pressure, temperature and salt concentration, two mass balance equations of solid phase and gas phase are solved.

The numerical simulation was conducted for a one-dimensional two-layered landfill with impermeable bottom and free-venting surface (Figure 3-4(a)). The boundary and initial conditions are listed in Equations (3-13a), (3-13b) and (3-16) with \( J_b=0 \). The lower layer contains a type of readily degradable waste. Compared with the waste from the lower part, the waste in the upper layer is fresh and looser with relatively higher gas conductivity. The parameters for two layers are shown in Figure 3-4(a). In the remainder of the paper, unless
noted specifically, the other default input parameters are the same as those in Table 3-1, which are within the typical ranges suggested from previous publications.

Comparisons between analytical solution given in this paper and numerical simulation by CODE_BRIGHT is shown in Figure 3-4(b) for both total average strain and excess gas pressure built up at the bottom. Total average strain mentioned here refers to the ratio of the total compression of the landfill over its initial height. It is a dimensionless indicator of the settlement happened at the landfill. As shown in Figure 3-4(b), the results given by two methods are very close to each other, which proves the correctness of the given solution.

(a) Free venting surface \( u_a = 0.0 \)

Upper layer
\[
\lambda = 8 \times 10^{-10} \text{s}^{-1}, \quad E_0 = 5000 \text{kPa}, \quad E = 200 \text{kPa} \\
\rho_0 = 760 \text{kg/m}^3, \quad k_a = 1.2 \times 10^{-6} \text{m/s}, \quad G_T = 220 \text{kg/m}^3
\]

Lower layer
\[
\lambda = 8 \times 10^{-8} \text{s}^{-1}, \quad E_0 = 1 \times 10^{20} \text{kPa}, \quad E = 1000 \text{kPa} \\
\rho_0 = 1000 \text{kg/m}^3, \quad k_a = 5 \times 10^{-7} \text{m/s}, \quad G_T = 100 \text{kg/m}^3
\]

Impermeable bottom

(b) Excess gas pressure and total average strain over time.
Table 3-1: Parameter values applied in the model.

<table>
<thead>
<tr>
<th>Name of parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of landfill</td>
<td>$H$</td>
<td>20</td>
<td>m</td>
</tr>
<tr>
<td>Initial porosity of waste</td>
<td>$\phi_0$</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Bulk density of waste</td>
<td>$\rho_0$</td>
<td>760</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Gas generation potential</td>
<td>$G_T$</td>
<td>157</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Gas conductivity</td>
<td>$k_a$</td>
<td>$8.6 \times 10^{-7}$</td>
<td>m/s</td>
</tr>
<tr>
<td>Reaction-rate constant of the waste</td>
<td>$\lambda$</td>
<td>$1.62 \times 10^{-9}$</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>Density of solid phase</td>
<td>$\rho_s$</td>
<td>1500</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Thickness of the cover</td>
<td>$d_l$</td>
<td>1.0</td>
<td>m</td>
</tr>
<tr>
<td>Gas conductivity of the cover</td>
<td>$k_l$</td>
<td>$8.6 \times 10^{-8}$</td>
<td>m/s</td>
</tr>
<tr>
<td>Residual modulus of solid skeleton</td>
<td>$E$</td>
<td>350</td>
<td>kPa</td>
</tr>
<tr>
<td>Primary modulus of solid skeleton</td>
<td>$E_0$</td>
<td>10000</td>
<td>kPa</td>
</tr>
</tbody>
</table>

*a* Value is within the range mentioned by Oweis et al. (1990).

*b* Value is measured by Nastev et al. (2001).

*c* Based on the typical value of gas conductivity and gas viscosity mentioned by Townsend et al. (2005).

*d* In the absence of site specific data, the default value suggested by Oweis (2006) which is equivalent to half life $t_{1/2}=-\ln(0.5)/\lambda=13.6$ year.

*e* The cover properties are based on the values used in Arigala et al (1995).

*f* Values are within the range summarized by Edil et al. (1990).
Time evolution of total average strain (Figure 3-4(b)) shows clearly two different stages. Most of the settlement that happened in the first stage (t<2000 days) comes from the secondary compression happened in the lower layer, and settlement in the later stage (t>2000 days) is mainly due to the compression of the fresh waste from the upper layer. The evolution of total average strain indicates that the proposed model is capable of considering different waste properties for each layer in a multi-layered landfill.

The gradual dissipation of excess gas pressure within the landfill is shown in Figure 3-4©. Gas flows upwards into the atmosphere and the maximum gas pressure built up at the bottom of the landfill. The abrupt transition in the gas pressure curve at the intersection of the two layers is caused by the distinct values of gas conductivity for two layers.

**3.4.2 Comparison with field measurements**

**3.4.2.1 Coll Cardús landfill**

Coll Cardús sanitary landfill is an active municipal and industrial solid waste disposal centre in the region of Catalonia, located in a natural valley in east-northern Spain. The landfill began in the mid of 1980s and the average depth of waste reaches about 60 m (Figure 3-5). Two groups of total four settlement control points are selected for the purpose of analysis. Group 1 includes A-2 and B-2, which are situated on the front slope and both are single-layered. Group 2 includes C-2 and E-1, which are located in the middle of the landfill and both have more than one layer of refuse. The detailed construction information at each point is listed in Table 3-2. The landfill was treated as impermeable at the base and free-venting at the top surface. Because there were no field records for gas flux or gas pressure, default value of $1.62 \times 10^{-9} \text{ s}^{-1} (0.05 \text{ year}^{-1})$ for reaction-rate constant suggested by Oweis et al. (1990) was adopted in the analysis.

Model simulations were performed using the analytical solutions given in the Appendix with the parameters listed in Table 3-1 & Table 3-2. Mechanical properties were adjusted in a reasonable range to best fit the field records. The simulated results in Figure 3-6 show a good agreement with field measurements at four settlement control points. This proves the ability of the proposed model in predicting settlement for both single-layered and multi-layered MSW landfills.
Table 3-2: Parameter values for Coll Cardús Landfill.

<table>
<thead>
<tr>
<th>Group</th>
<th>Control points</th>
<th>Layer No.</th>
<th>Constructed time (year)</th>
<th>Layer height (m)</th>
<th>$E_0$ (kPa)</th>
<th>$E$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>A-2</td>
<td>1</td>
<td>1997</td>
<td>22.1</td>
<td>10000</td>
<td>550</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>1</td>
<td>1997</td>
<td>18.6</td>
<td>3000</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>C-2</td>
<td>1</td>
<td>2001</td>
<td>8.43</td>
<td>10000</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1992</td>
<td>32.7</td>
<td>$1 \times 10^{20}$</td>
<td>700</td>
</tr>
<tr>
<td>Group 2</td>
<td>E-1</td>
<td>1</td>
<td>2002</td>
<td>9.22</td>
<td>10000</td>
<td>350</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1998</td>
<td>12.6</td>
<td>$1 \times 10^{20}$</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>1992</td>
<td>32.2</td>
<td>$1 \times 10^{20}$</td>
<td>1000</td>
</tr>
</tbody>
</table>
Figure 3-6: Comparison between field measurements and model simulation for Coll Cardús landfill.
### 3.4.2.2 Mountain View Controlled landfill

Large-scale test was conducted under different operational management practices at the Mountain View Controlled landfill in north-east of California, U.S.A by El-Fadel et al. (1996 & 1999). The test included six cells and each cell was 30 m in length, 30 m in width and 15 m in depth. All the six cells were filled with municipal solid refuse which was placed in the cell at the same time, and each cell was lined with a 1.5 m thick compacted clay liner at the base as well as at the side wall. The bulk density recorded in the test is 704 kg/m³. The gas mixture flowing out of each cell was collected and measured at the top and cell settlement was monitored at the cell surface over a period of 4 years. Among all the six cells, cell F was arranged to be the control cell. The averaged gas flux measured at the top surface of cell F is about $1.23 \times 10^{-5}$ kg/m²/s (Figure 3-7(a)) and the maximum vertical strain at the end of 1576 days measured at cell F reaches 13% (Figure 3-7(b)).

Table 3-3: Parameter values for Mountain View Controlled landfill.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>15</td>
<td>m</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>704</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$3 \times 10^{-9}$</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>$G_T$</td>
<td>330</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$E$</td>
<td>150</td>
<td>kPa</td>
</tr>
<tr>
<td>$E_0$</td>
<td>20000</td>
<td>kPa</td>
</tr>
</tbody>
</table>

Figure 3-7: Comparison between field measurements and model simulation at Mountain View Controlled landfill (a) gas flux at top surface. (b) Total average strain.
Simulation of both settlement and gas flow rate was carried out for cell F using the proposed model. The cell was treated as impermeable at the base and free-venting at the top surface. Parameter values are listed in Table 3-3 with other default values in Table 3-1. Reaction-rate constant and mechanical properties were adjusted within the reasonable range to best fit test measurements.

Simulated gas flux at the top surface decreases with time due to first order kinetic gas generation law. The averaged value during the first 2000 days agrees with the field records (Figure 3-7(a)). Simulated total average strain fits well with field measurements as shown in Figure 3-7(b). The results of the comparison prove that the proposed model is capable of predicting both the settlement and gas production for MSW landfills.

3.5 Evaluation of Parameters

Based on the solutions given in appendix 3-A, influences of various operating scenarios and material properties on landfill behaviour are investigated in this section for a single-layered landfill to gain insight into the characteristics of settlement evolution and gas pressure distribution. Default parameter values are listed in Table 3-1 unless noted specifically.

3.5.1 Effects of different operating scenarios

Distribution of excess gas pressure under different operating scenarios is presented in Figure 3-8. Base case represents a landfill with free-venting top surface and impermeable bottom. For case 1, the specified flux rate at the bottom, $J_b$, is taken to be $-1.32 \times 10^{-5}$ kg/m$^2$/s (negative means influx), which is equivalent to the gas emission rate from a landfill with height of 30 m (based on the recommended gas generation rate of $4.4 \times 10^{-7}$ kg/m$^3$/s mentioned by Townsend et al., 2005). In case 2, gas collection is implemented at both top surface and bottom with vacuum pressure of -2 kPa. Case 3 is used for illustrating the effect of a low-permeable top cover on the distribution of excess gas pressure.

Results show that in the base case, gas flows upward into the atmosphere and the highest gas pressure about 0.5 kPa was built up at the bottom. Much higher gas pressure built up in case 1 indicates gas pressure at the leachate level in a high landfill increases dramatically if there is no other gas extraction system available. While in case 2, the application of vacuum of -2 kPa at both ends results in negative gas pressure all over the landfill. For a covered landfill as in case 3, excess gas pressure builds up beneath the cover. The comparison between the base case and case 3 shows that the cover retards the gas flux at the top surface; thus, the gas pressure within the landfill is higher than the case without cover.
3.5.2 Evolution of bulk density

The temporal and spatial density variation of bulk density in a single-layered, covered landfill (case 3) is examined in this section. The initial bulk density and porosity are supposed to be homogeneous within the landfill. The vertical stress is equal to the gravitational stress for the closed landfill. Time evolution of the bulk density at different depth in the landfill is shown in Figure 3-9. Density exhibits different evolution trend for waste from different depth. The bulk density for the waste in the lower part increases continuously with time because the mechanical compression is larger than void enlargement due to mass degradation. The decreased bulk density with time in the upper part indicates that the newly formed void volume is greater than the void decreasing due to skeleton compression owing to the less stress level in the upper part of the landfill.

3.5.3 Effect of gas conductivity

The influence of gas conductivity on the settlement and gas pressure is examined for case 3. Typical values of gas conductivity for the waste are within the range from $1.16 \times 10^{-7}$ to $1.74 \times 10^{-6}$ m/s based on the report of Lang and Tchobanoglous (1989). Result in Figure 3-10 clearly shows that the excess gas pressure within the landfill decreases with increasing value of $k_a$. Settlement curves demonstrate that within the typical range of $k_a$, the influence of gas
conductivity on settlement behaviour can be ignored because excess gas pressure built up within the landfill is only a few kPa.

![Figure 3-9: Evolution of bulk density with time at different depth of the landfill.](image)

![Figure 3-10: The effect of gas conductivity, ka, on the time evolution of total average strain and excess gas pressure at the bottom of the landfill.](image)

### 3.5.4 Effect of cover properties

The solution for case 3 (Appendix 3-A) indicates that the effect of cover properties is determined by an integrated parameter, Lc, defined as \( k_i/(d_i/k_a) \). Lower value of Lc can be obtained by increasing the thickness of the cover or decreasing the gas permeability of the cover material. The dependence of total average strain and excess gas pressure on Lc (case 3) is shown in Figure 3-11 at the time of 5 years after closure. Variable values of Lc are
obtained through setting \( k_t/k_a = 0.1 \) and varying \( d_t \) with all the other input parameters remaining the same as those in Table 3-1.

Smaller value of \( L_c \) (less permeable of the cover) causes larger gas pressure due to more tardiness for the gas flowing outwards through the top cove. Settlement rate of the landfill is delayed correspondingly due to the correspondingly smaller effective stress. When \( L_c > 0.1 \), which is equivalent to a 1-meter-thick cover with gas conductivity of one order less than gas conductivity of the waste, the top cover will no longer affect the behaviour of a 20-meter-high landfill after 5 years of closure.

![Graph showing impact of cover parameter, \( L_c \), on the total average strain and excess gas pressure at the bottom of the landfill, 5 years after closure.](image)

**Figure 3-11: Impact of cover parameter, \( L_c \), on the total average strain and excess gas pressure at the bottom of the landfill, 5 years after closure.**

### 3.6 Conclusions

Analytical solutions to predict one-dimensional settlement and gas flow for both single-layered and multi-layered MSW landfills were presented. Mechanical compression of the solid skeleton is coupled with gas pressure using K-H viscoelastic model. The solutions of settlements as well as excess gas pressure within the landfill were obtained analytically in the Laplace transform domain. The solutions were verified by comparison with numerical simulation. The comparisons with field records at two landfill sites prove that the proposed model can well reproduce the time evolution of settlement and predict gas flux in the horizontal LFG collection systems.
Analytical solution always serves as a useful and convenient tool for parametric studies. The effects of gas conductivity, $k_a$, and cover property, $L_c$, on the rate of settlement and gas pressure were examined. The results show that the rate of settlement is not so much influenced by $k_a$ and $L_c$, although excess gas pressure is sensitive to these parameters. In the typical MSW landfill, excess gas pressure is of several orders less than gravitational stress and the coupling effect between gas pressure and mechanical compression is too little to be noticeable. The coupling effect may become apparent under the cases of low gas conductivity, deep landfills, bioreactor landfills, or inefficient gas extraction system. Under these circumstances, the increased gas pressure may apparently delay the settlement of the landfill.

The model presented in this paper focuses only on solid phase and gas phase. It is applicable for closed-landfill or waste part above leachate level. This model will be extended to include liquid phase and consider the interaction among solid, gas and liquid phases in the future.
Appendix 3-A

The solutions of excess gas pressure within the landfill at depth of \( z \), \( u_a(z) \), total compression of the waste column, \( S \), and gas flux at the top, \( J_t \), of one-layered landfill for three different boundary scenarios listed in Equations (3-13)-(3-16) are given in the Laplace transform domain as follows:

1. The solutions for case 1 are

\[
\tilde{u}_a(z) = \left( \frac{C_d}{s(s + \lambda)} + \frac{C_{uo}u_{o0}}{s} \right) \left[ 1 - \frac{\exp(-\chi(H - z)) + \exp(-\chi z)}{1 + \exp(-2\chi H)} \right] \\
- \left( \frac{C_v}{\chi s} + \frac{J_k g}{\chi \lambda k_a} \right) \frac{\exp(-\chi(H - z)) - \exp(-\chi(z + H)) + C_v z}{1 + \exp(-2\chi H)} + \frac{C_v}{s} z
\]

\[
\tilde{S} = \frac{\tilde{m}_a}{s} = \left\{ \frac{\rho_ogH^2}{2} + u_{a0}H - \left( \frac{C_d}{s + \lambda} + \frac{C_{uo}u_{o0}}{s} \right) \left[ \frac{H + \frac{\exp(-2\chi H) - 1}{\chi(1 + \exp(-2\chi H))}}{1 + \exp(-2\chi H)} \right] \right\} \\
+ \left( \frac{C_v}{\chi s} + \frac{g J_k}{k_a} \right) \left[ \frac{(1 - \exp(-\chi H))^2}{\chi^2(1 + \exp(-2\chi H))} \right] - \frac{C_v}{\chi^2(1 + \exp(-2\chi H))} \frac{H^2}{2}
\]

\[
\tilde{J}_t = -\frac{k_a}{g} \left( \frac{C_d}{s + \lambda} + \frac{C_{uo}u_{o0}}{s} \right) \chi \left[ 1 - \frac{\exp(-2\chi H)}{1 + \exp(-2\chi H)} \right] - \frac{1}{s} \left( \frac{C_v}{s} + \frac{g J_k}{k_a} \right) \left[ \frac{2\exp(-\chi H)}{1 + \exp(-2\chi H)} \right] + \frac{C_v}{s}
\]

where \( \chi = \sqrt{\frac{s}{C_v}} \).

2. The solutions for case 2 are

\[
\tilde{u}_a(z) = \left( \frac{C_d}{s(s + \lambda)} + \frac{C_{uo}u_{o0}}{s} \right) \left[ \frac{\exp(-\chi(H - z)) - \exp(-\chi(2H - z)) + \exp(-\chi z) - \exp(-\chi(H + z)) + 1}{\exp(-2\chi H) - 1} \right] \\
+ \frac{u_a}{s} \frac{\exp(-\chi(H + z)) - \exp(-\chi(H - z))}{\exp(-2\chi H) - 1} + \frac{u_t}{s} \frac{\exp(-\chi(2H - z)) - \exp(-\chi z)}{\exp(-2\chi H) - 1} \\
+ \frac{C_v}{s} \left( \frac{\exp(-\chi(H - z)) - \exp(-\chi(H + z)) + z}{\exp(-2\chi H) - 1} \right)
\]
\[
\tilde{S} = \frac{\tilde{m}_s^*}{s} \left[ \frac{\rho_0 g H^2}{2} + u_{sh} H - \frac{C_d}{s + \lambda} + C_{wsh} u_{sh} \right] H - C_v \frac{H^2}{2} \\
- \left[ 2 \left( \frac{C_d}{s + \lambda} + C_{wsh} u_{sh} \right) - u_b - u_v + C_v H \right] \frac{(1 - \exp(-\chi H))^2}{\chi(\exp(-2\chi H) - 1)}
\]

\[
\tilde{J}_v = -\frac{k_s}{g} \left[ \frac{\chi}{s} \left( \frac{C_d}{s + \lambda} + C_{wsh} u_{sh} \right) \frac{-L_v \exp(-2\chi H) + L_v (L_v - \chi) \exp(-2\chi H) + L_v + \chi}{(L_v - \chi) \exp(-2\chi H) + L_v + \chi \chi} \right] + \frac{C_v}{\chi} \frac{-L_v (1 - \exp(-\chi H))^2}{(L_v - \chi) \exp(-2\chi H) + L_v + \chi} \frac{H^2}{2}
\]

3. The solutions for case 3 are

\[
\tilde{u}_v(z) = \left( \frac{C_d}{s + \lambda} + C_{wsh} u_{sh} \right) \left[ 1 - \frac{L_v \exp(-\chi (2H - z)) + L_v \exp(-\chi z)}{(L_v - \chi) \exp(-2\chi H) + L_v + \chi} \right] + \frac{C_v}{s} \left[ \frac{\exp(-\chi (2H - z)) - L_v \exp(-\chi (H - z)) + \exp(-\chi z) + \frac{L_v - \chi \exp(-\chi (z + H))}{(L_v - \chi) \exp(-2\chi H) + L_v + \chi}}{\chi} \right] + z
\]

where \( L_v = \frac{k_v}{d_k u_a} \).
Appendix 3-B

The transfer matrices at depth \( z (Z_{j+1} < z < Z_j) \) within \( j \)th layer are expressed as

\[
T_j(\xi) = \begin{pmatrix}
ch\lambda_j \xi & -\frac{g}{k_{aj}} \text{sh}\lambda_j \xi \\
-\frac{s}{V_{v_j} \lambda_j} \text{sh}\lambda_j \xi & ch\lambda_j \xi
\end{pmatrix}
\]

\[
S_j(\xi) = \begin{pmatrix}
\frac{C_{aj}}{V_{v_j}} \frac{1-ch\lambda_j \xi}{s+\lambda_j} + \frac{C^z_{v_j}}{V_{v_j}} \left( \frac{\xi}{\lambda_j} - \frac{\text{sh}\lambda_j \xi}{\lambda_j^2} \right) \frac{1}{\lambda_j^2} \\
\frac{C_{aj}}{V_{v_j}} \frac{1-\text{sh}\lambda_j \xi}{s+\lambda_j} - \frac{C^z_{v_j}}{V_{v_j}} \left( \frac{1-ch\lambda_j \xi}{\lambda_j^2} \right) \frac{k_{aj}}{g}
\end{pmatrix}
\]

where \( \lambda_j = \sqrt{-\frac{s}{C^a_{v_j}}} \) and \( \xi \) is the depth from the top surface and equal to \( z - Z_{j+1} \).

The solutions for the vector of excess gas pressure and gas flux at the top surface of the landfill in the Laplace transform domain are

For scenario 1: \( \tilde{\mathbf{X}}(0) = \begin{pmatrix} \tilde{u}_a(0) \\ \tilde{J}(0) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{J_b}{TT_{22}} \end{pmatrix} \)

For scenario 2: \( \tilde{\mathbf{X}}(0) = \begin{pmatrix} \tilde{u}_a(0) \\ \tilde{J}(0) \end{pmatrix} = \begin{pmatrix} \frac{u_a}{s} + \frac{1}{TT_{12}} \frac{u_b}{s} - SS_1 TT_{12} \\ -TT_{11} \frac{u_a}{s} + \frac{1}{TT_{12}} \frac{u_b}{s} - SS_1 TT_{12} \end{pmatrix} \)

For scenario 3: \( \tilde{\mathbf{X}}(0) = \begin{pmatrix} \tilde{u}_a(0) \\ \tilde{J}(0) \end{pmatrix} = \begin{pmatrix} \frac{SS_2}{TT_{22}} k_{ai} d_i \\ -\frac{SS_2}{TT_{22}} k_i \end{pmatrix} \)

where \( TT_{ij} \) and \( SS_i \) are components of matrices \( \mathbf{T}(Z_n) \) and \( \mathbf{S}(Z_n) \):

\[
\mathbf{T}(Z_n) = \begin{bmatrix} TT_{11} & TT_{12} \\ TT_{21} & TT_{22} \end{bmatrix} = \mathbf{T}_n(H_n) \cdots \mathbf{T}_2(H_2) \mathbf{T}_1(H_1)
\]

\[
\mathbf{S}(Z_n) = \begin{bmatrix} SS_1 \\ SS_2 \end{bmatrix} = \sum_{i=1}^{n} \mathbf{T}_i(H_n) \cdots \mathbf{T}_{i+1}(H_{i+1}) \mathbf{S}_i(H_i)
\]
CHAPTER 4

Gas flow to a vertical gas extraction well in deformable MSW landfills

Abstract

Active gas control systems are commonly used in municipal solid waste (MSW) landfills and the design of such systems requires thorough understanding of the gas flow pattern. A model is developed to predict the two-dimensional radial transient gas flow to a vertical gas extraction well in deformable MSW landfills. Variations of gas storage include time-dependent compression of the refuse, dissolution of gas components and porosity enlargement due to organic matter degradation. Mechanical compression of solid skeleton is coupled with gas pressure using K–H rheological model which is capable of reproduce the evolution of settlement for MSW landfills. The new analytical solution obtained in Laplace transform domain can be used to determine excess gas pressure fields, gas fluxes in the well and through the top cover as well as landfill settlements. The solution is validated by comparison with field measurements and numerical simulations. It demonstrates that the gas storage variation term becomes predominant only during early times. Long-term gas flow is controlled by the gas generation rate and the quasi-steady solution is valid. Parametric studies indicate that the solution given in this paper is useful for the prediction of gas fluxes, for the choice of the optimum spacing between wells, and for the determination of the final cover properties as well as appropriate vacuum pressure imposed in the extraction well.

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Key words: Municipal solid waste landfill; Extraction well; Gas flow; Laplace transform; Analytical method

4.1 Introduction

Sanitary landfilling is generally believed to be the most essential and economic method for the disposal of municipal solid waste (MSW). Once the waste is deposited, landfill gas generates continuously through anaerobic biodegradation of the organic fraction of waste. Landfill gas (LFG) typically contains 50% methane (CH$_4$). Most of the remainder is carbon dioxide (CO$_2$). Both gases are greenhouse effect gases, but methane is so powerful that its emissions from MSW are a source of concern for global change. Methane is also feared to increase the risk of explosion from localized build up. Moreover, landfill gas contains toxic contaminants that may cause cancer and other health problems in local communities. As a result, the migration of landfill gas should be carefully controlled. Active LFG control systems are extensively used for this purpose. They typically involve a network of vertical wells distributed throughout the landfill and subsequent vacuum to extract the biogas.

Understanding the mechanisms that explain LFG migration is essential for the design and management of LFG control systems. Several distinct features distinguish refuse in MSW landfills from other geomaterials. First, the refuse is heterogeneous not only on its mechanical, hydraulic and transport properties, but also on its biochemical properties. Since full characterization is virtually impossible, efforts must be made to seek effective parameters and models, which capture the essential aspects of the waste, while acknowledging their approximate nature. Second, biogas is continuously generated through the progress of biochemical reactions. Third, gas filled porosity evolves over time in response to both stress-induced compression and enlargement due to solids loss. Therefore, the model used to simulate gas flow in the MSW landfills should be capable of considering these processes.

Extensive analytical solutions have been developed to model gas movement towards an extraction well for problems of subsurface contamination remediation. A quasi-analytical solution for radial transient gas flow to a well in vadose zone was developed within a horizontally infinite domain by McWhorter (1990), who used a linear elastic model to account for compressibility of the medium. Baehr and Hult (1991) produced analytical solutions for two-dimensional steady-state gas flow toward a partially penetrating well, which are useful for the design of soil vapour extraction schemes because transient flow effects dissipate quite rapidly. However, these solutions are of limited applicability to MSW problems because they lack a gas generation term.
Several analytical solutions describing gas flow around extraction wells in MSW landfills have been developed for steady state based on the assumption that the landfill is rigid and gas generation rate is constant. Young (1989) obtained an analytical solution to a two-dimensional transport problem in a rectangular cross-section when a series of horizontal extraction wells are present. Arigala et al. (1995) extended the solution by Young (1989) to three-dimensions with multiple vertical wells placed at arbitrary points. For the sake of mathematical convenience, solutions are given under steady-state condition and the wells are simplified as line sinks with certain gas-withdrawal rate along well depth in these analytical models.

In the numerical modelling of the two-dimensional gas flow around a single well, Chen et al. (2003) and Martín et al. (2001) analyzed gas flow around a passive and an active well in rigid MSW landfill, respectively, using the finite difference method while neglecting early time variations in gas storage in the porous medium. Some other numerical models have been developed to incorporate more complex conditions for gas transport in MSW landfills. Nastev et al. (2001) conducted a coupled thermo-hydro-gas (THG) analysis which considers multi-component gas migration in unsaturated soil. A three dimensional model of multi-component gas generation and transport with arbitrary number of gas extraction wells was developed by Hashemi et al. (2002) and Sanchez et al. (2006) under quasi-steady-state and dynamic conditions, respectively.

A landfill is a multiphase medium with each phase exhibiting significant spatial and temporal variations (Durmusoglu et al., 2005). The majority of gas flow models in MSW landfills focused only on gas phase without taking into account the variation of mass storage of gas with time in the landfill. Recently several published works have proposed some models that could describe gas migration in a deformable MSW landfill. In the absence of analytical solutions, Hettiarachchi et al. (2007) developed a computer program to numerically predict gas pressures and settlements based on the mass balance of gas. The laboratory compression curve served as the constitutive law for stress-induced deformation in the landfill. Liu et al. (2006) coupled excess gas pressure and mechanical deformation based on the linear unsaturated consolidation theory by Fredlund and Rahardjo (1993). Both models assumed that the variation of medium porosity is the sum of compressibility induced by the variations of effective stress and the enlarged void volume due to degradation. Whether the newly formed void volume due to biodegradation will totally transform into landfill compression is questionable. Durmusoglu et al. (2005) derived a multiphase mathematical model considering solid-gas-liquid interactions, in which Maxwell viscoelastic model was used to model the porosity variation. Based on Maxwell’s viscoelastic model, the system will have infinite deformation under effective stress, which is not the case in the landfill. None of these models account for the fact that gas pressure variations will lead to variations in the dissolved fraction of each gas component, which may effectively cause a large increase in gas storativity.
In this paper, a model was developed to simulate the unsteady gas flow around a vertical extraction well in deformable MSW landfills. The model considers both exponentially decaying gas generation and the variation of mass storage of gas due to time-dependent compression of the refuse, dissolution of gas components and porosity enlargement. The solutions to the proposed model for excess gas pressures, gas fluxes in the well and through the top cover, as well as settlements, were obtained analytically.

4.2 Model development

Figure 4-1(a) shows the schematic representation of the problem under consideration. A fully penetrating vertical well of radius \( r_w \) is located in the centre of a cylindrical refuse cell bounded by an outer radius \( R_e \). The cylindrical cell typically represents one of many in a network of extraction wells where wells are equidistant from each other and \( R_e \) represents the half distance between two neighbouring extraction wells (Figure 4-1(b)). The landfill has a thickness of \( H \), a low-permeable cover at the top and a fixed flux at the bottom to simulate an influx of gas from a possible saturated layer at the base (Young 1989). Fixed vacuum pressure is prescribed along the extraction well, and the radial outer boundary is treated to be impermeable. Although the cylindrical cell is considered as somewhat of an idealization of the real problem, it does however greatly facilitate the mathematical analysis without unduly affecting the overall accuracy (Chin 2004). The basic assumptions made in developing the model in the present paper are:

1) All compressive strains within the landfill occur in vertical direction.
2) Darcy’s law is obeyed and gas follows the ideal gas law.
3) Dissolution of gas components obeys Henry’s law.

4.2.1. Gas generation

The rate of gas generation is approximated as a generic monotonically decreasing function

\[
\alpha = G_T \sum_{i=1}^{N} A_i \lambda_i e^{-\lambda_i t'}
\]  

(4-1)

where \( \alpha \) is the overall gas production rate (kg/m\(^3\)/s); \( G_T \) is total potential for gas generation per unit volume of refuse (kg/m\(^3\)); \( t' \) is time elapsed since waste deposition (s); \( A_i \) and \( \lambda_i \) are used to approximate \( \alpha \). In practice they can be viewed as categories of waste displaying a first order degradation behaviour, in which case \( A_i \) is the proportion of category \( i \) and \( \lambda_i \) is its
degradation rate constant ($s^{-1}$). This expression is similar to that first proposed by Findikakis and Leckie (1979), who assumed that the waste can be broadly categorized into three representative categories: readily, moderately and slowly biodegradable waste (i.e., $N=3$).

Figure 4-1: (a) Schematic representation of the cylindrical domain used to model gas flow around a vertical gas extraction well. (b) Network of extraction wells uniformly distributed in the landfill.
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As a real landfill usually takes many years to fill, refuse at the bottom of the landfill is expected to have a different gas production rate from the refuse close to the top surface. To account for the different gas production rate at different depth in a landfill with linearly depositing rate, time \( t' \) in Equation (4-1) is defined by (Arigala et al., 1995)

\[
t' = t + \frac{z}{H} t_f
\]

where \( t \) is time elapsed since landfill closure (s); \( t_f \) is the time it took to fill the landfill (s); \( z \) is depth (m); \( H \) is the final thickness of the landfill (m).

### 4.2.2 Mass storage of gas

Gaseous phase in the landfill is considered as an approximately fixed mixture of gaseous components (here, we will assume these to be \( \text{CH}_4 \) and \( \text{CO}_2 \)) which are also dissolved in aqueous phase. The solubility of gas is very sensitive to pH value of leachate which normally varies between 6 and 8.5 in a typical MSW landfill. \( \text{CO}_2 \) turns to be highly soluble under alkaline conditions (approximately an increase of a factor of 10 per unit increase in pH). The total amount of gaseous components in the porous medium is expressed by

\[
M_g = \theta_g \rho_g + \theta_l \sum_{i=1}^{2} c_i
\]

where \( M_g \) is the total mass of gas per unit volume of porous medium (kg/m\(^3\)); \( \theta_g \) is the volumetric gas content which is equal to \( \phi(1 - S_l) \); \( \phi \) is the porosity and \( S_l \) is degree of liquid saturation defined as the volumetric fraction of voids occupied by liquid phase; \( \rho_g \) is the density of gas (kg/m\(^3\)); \( \theta_l \) is the volumetric moisture content which is equal to \( \phi S_l \); \( c_i \) is the concentration of the \( i \)th gas component dissolved in the aqueous phase (kg/m\(^3\)). Henry’s law gives

\[
P_{g,i} = K_{H,i} c_i
\]

where \( P_{g,i} \) is the partial pressure of the \( i \)th gas component (kPa); \( K_{H,i} \) is Henry’s constant of \( i \)th gas component and is expressed in units of pressure per units of concentration in the liquid phase (kPa·m\(^3\)/kg). If we assume that gas pressure fluctuations do not affect significantly the mixing ratio of each gas component, then
\[
\sum_{i=1}^{2} c_i = P_g \sum_{i=1}^{2} \frac{x_i}{K_{H,i}} = \frac{P_g}{K_{Hg}} \tag{4-5a}
\]

\[
\frac{1}{K_{Hg}} = \frac{x_{CO_2}}{K_{H,CO_2}} + \frac{x_{CH_4}}{K_{H,CH_4}} \tag{4-5b}
\]

where \(P_g\) is absolute gas pressure and is expressed as \(P_g = u_g + u_{atm}\), \(u_{atm}\) is unit atmospheric pressure (101.3 kPa), and \(u_g\) is excess gas pressure (kPa); \(x_i\) is the molar fraction of \(i\)th component in the gas phase; \(K_{Hg}\) is an “apparent” Henry’s constant which is determined by the Henry’s constants of gaseous components as well as their molar fractions. \(K_{Hg}\) takes the value of 180 kPa·m\(^3\)/kg for equimolar condition \(x_{CO_2} = x_{CH_4} = 0.5, K_{H,CO_2} = 91.3\) kPa·m\(^3\)/kg, \(K_{H,CH_4} = 5562.5\) kPa·m\(^3\)/kg).

The ideal gas law is used to describe the relation between gas density and absolute gas pressure

\[
\rho_g = \frac{P_g M}{RT} \tag{4-6}
\]

where \(R\) is universal gas constant (0.008314 kJ/mol/K); \(T\) is absolute temperature of gas (310K when the effect of temperature variation is negligible); \(M\) is average molecular weight of gas (kg/mol), here it is assumed that the average molecular weight of gas in the process of biodegradation does not change, and \(M\) is taken to be 0.03 kg/mol.

Substituting Equations (4-4), (4-5a) and (4-6) into Equation (4-3), and rewriting volumetric moisture and gas content with porosity, the total amount of gaseous components per unit volume of porous medium is

\[
M_g = \xi \phi \rho_g \tag{4-7a}
\]

\[
\xi = (1 - S_i) + S_i \frac{1}{K_{Hg}} \frac{RT}{M} \tag{4-7b}
\]

where the new parameter \(\xi\) is defined as volumetric fraction of gaseous components (i.e., the volume these components would occupy if they were totally in the gas phase, divided by the volume of voids). The value of \(\xi\) depends on degree of liquid saturation and “apparent” Henry’s constant, \(K_{Hg}\). High value of \(1/K_{Hg}\) means the solubility of the gaseous components is high.
The hydraulic conductivity of refuse is of the order of $10^{-5}$ m/s for a 40-meter high landfill according to the hydraulic conductivity curve by Powrie et al. (1998) based on a series of large-scale compression tests of MSW. Residual saturation will be retained within the refuse of depth less than 40 m in the normal landfill because the free water will drain out quickly. Or a target degree of saturation is maintained in modern bioreactor landfills by re-circulating the leachate collected from the bottom of the landfill. Thus it is convenient to consider the refuse in the upper part of the landfill with a constant saturation.

Using Equation (4-7a) variations in mass storage will be given by

$$\frac{\partial M_g}{\partial t} = \zeta \rho_g \frac{\partial \phi}{\partial t} + \zeta \phi \frac{\partial \rho_g}{\partial t}$$  \hspace{1cm} (4-8a)

Considering that porosity may change because of both compression of the matrix and solid degradation, while using Equation (4-6) for $\rho_g$, allows us to write

$$\frac{\partial M_g}{\partial t} = \zeta \rho_g \left( \frac{\partial \varepsilon_s}{\partial t} + \frac{Y \alpha}{\rho_s} + \frac{\phi}{\rho_g} \frac{\partial \rho_g}{\partial t} \right)$$  \hspace{1cm} (4-8b)

where $\varepsilon_s$ is volumetric strain of solid; $\rho_s$ is the density of solid phase (kg/m$^3$); $Y$ is gas yield coefficient of refuse (mass of solid phase degraded/mass of gas phase generated). The term of $Y \alpha / \rho_s$ represents the rate of volume change due to degradation of the organic matter.

### 4.2.3. Mechanical deformation

Continuous degradation of solid matter distinguishes MSW from ordinary soil-like materials. Waste exhibits a strong time-dependent behaviour. There is no rigorous model describing the relationship between the enlarged void volume due to solids degradation and the corresponding compressive strain. McDougall and Pyrah (2004) proposed a new parameter $\Lambda$, degradation-induced void change parameter, to describe the relationship between decomposed solid phase, $dV_s$ and the induced change in void volume, $dV_v$ as $dV_v = \Lambda dV_s$. The authors also indicated that the value of $\Lambda$ is not constant but changes with the progress of degradation. When $\Lambda=0$, the newly formed void volume due to degradation totally transforms into compression of the landfill, which was the assumption of Hettiarachchi et al.(2007) and Liu et al. (2006). From the mechanical point of view, mass loss does not induce settlement itself. The newly formed void volume resulting from solids degradation does not translate to subsidence directly. Instead, mass loss causes porosity of the waste to increase, and stiffness of solid matrix to decrease. That is, biodegradation of solid mass weakens the
micro-structure of waste through conversion of the solid mass into biogas. The subsequent settlement is the result of collapse of the weakened waste skeleton under stress.

The compressibility of solid is controlled by two stress invariants (Fredlund and Rahardjo, 1993): net normal stress, \( \sigma - u_g \), and matric suction, \( u_g - u_{water} \), where \( \sigma \) is the total stress, \( u_g \) and \( u_{water} \) are excess gas pressure and water pressure, respectively. When there is no external load applied on the landfill, the total stress inside the refuse will be gravitational stress caused by self-weight.

Based on the study from Yuen and Styles (2000), the mean value of moisture content is 55% (dry mass basis) which is equivalent to 28% volumetric moisture content if bulk density of waste is considered to be 800 kg/m\(^3\). According to the retention curve measured by Kazimoglu et al (2005), the suction of the refuse is about 5kPa. Therefore the compressive strain of solid skeleton due to matric suction is negligible compared to that caused by net normal stress.

In this paper, the process of degradation and collapse is represented by a K–H rheological model (VOIGT form of the standard linear solid model as illustrated in Figure 3-2), which reflects both the time-dependent and stress-dependent deformation, and requires limited parameters which are easy to determine. The response of settlement to the void enlargement may be locally erratic. However, a smoothed time–strain curve is capable of describing the overall behaviour of settlement. In K–H rheological model, the first Hookean spring with a stiffness of \( E_0 \) is used to simulate the magnitude of compression occurring prior to biodegradation. Another Kelvin element consisting of two parallel basic elements, namely the second Hookean spring, characterized by its modulus \( E \), and a dashpot, characterized by its viscosity \( \eta \) is used to simulate the compression occurring during biodegradation.

The expression for volumetric strain can be split between that \( \varepsilon_1 \), corresponding to the Hookean spring of stiffness \( E_0 \), and \( \varepsilon_2 \), corresponding to the Kelvin element. There satisfy

\[
\varepsilon_1 = \sigma' / E_0 \\

\sigma' = E \varepsilon_2 + \eta \dot{\varepsilon}_2
\]

(4-9a)  (4-9b)

where \( \sigma' \) is effective stress (kPa); \( E_0 \) is elastic modulus of waste for primary settlement (kPa); \( E \) is residual elastic modulus of the solid matrix for secondary settlement (kPa); \( \eta \) is viscosity of solid skeleton (kPa·s).

These equations are best solved by employing Laplace transform. Transforming Equations (4-9a) and (4-9b), and adding the corresponding transformed strain, while using \( \sigma' = \rho_0 gz - u_g \), leads to
\[ \tilde{\varepsilon}_v = m_k^a \tilde{\sigma}' = -\left( \frac{1}{E_0} + \frac{1}{E + s\eta} \right) \left( \frac{\rho_0 g z}{s} - \tilde{u}_g \right) \]  

(4-10)

where \( \tilde{\varepsilon}_v = \int_0^\infty \varepsilon_v(t) e^{-\mu t} dt \) is Laplace transform of the volumetric strain; \( m_k^a \) is Laplace transform of the compression coefficient of solid skeleton (kPa\(^{-1}\)); \( \tilde{\sigma}' \) and \( \tilde{u}_g \) are Laplace transforms of effective stress and excess gas pressure (kPa); \( \rho_0 \) is bulk density of refuse (kg/m\(^3\)). Here the increase of pore volume is assumed to be positive, thus the coefficient \( m_k^a \) is negative.

The observed response of settlement in landfill displays a very close relationship with the progress of biodegradation. Settlement is controlled by the interaction of soil skeleton strength and stress state (McDoughall and Pyrah, 2004). If parameter \( Y \) is defined as gas yield coefficient of refuse (mass of solid phase degraded/mass of gas phase generated), \( YG_T \) is total solids loss per unit volume of refuse (kg/m\(^3\)). According to K–H viscoelastic model, the degradation-induced volume change at time \( t \) is \( YG_T (1 - \exp(-\lambda t)) / \rho_s \) and the compression of the solid matrix is \( f(\sigma')(1 - \exp(-Et/\eta))/E \). If it is assumed that the stiffness of the solid matrix decreases at the same rate with the degradation of solid mass, there is \( E/(\eta \lambda) = 1 \). Therefore the parameter \( \eta \) can be determined by the residual modulus and the degradation rate constant of the waste.

**4.2.4 Mass balance equation**

Under isothermal conditions, gas flow in porous media is governed by the mass balance equation

\[ \frac{\partial (M_g)}{\partial t} = -\nabla \cdot J_g + \alpha \]  

(4-11)

where \( M_g \) is the total mass of gaseous components (kg/m\(^3\)); \( J_g \) is the flux of gas phase (kg/m\(^2\)/s); \( \alpha \) is source term equal to the gas production rate as described in Equation (4-1) (kg/m\(^3\)/s). Under axisymmetric conditions, gas flux may be written as

\[ J_g = \begin{bmatrix} J_v \\ J_h \end{bmatrix} = \begin{bmatrix} k_v \left( \frac{\partial u_v}{\partial z} - \rho_g g \right) \\ k_h \frac{\partial u_g}{\partial r} \end{bmatrix} \]  

(4-12)
where $J_v$ and $J_h$ are the fluxes of gas in the $z$ and $r$ directions respectively (kg/m$^2$/s); $k_v$ and $k_h$ are gas conductivities in vertical and horizontal directions, respectively (m/s), and the landfill permeability has been considered to be anisotropic because of the layer-like deposition of the waste; $g$ is gravitational acceleration (9.807$m$/s$^2$). Comparing with gas flux under pressure gradient, gas flux due to gravitational gradient normally can be neglected.

Using Equation (4-8b) for $\partial M_g / \partial t$, Equation (4-12) for $J_g$ and neglecting gas flux due to gravitational gradient, Equation (4-11) can be rewritten as

$$
\frac{\xi \rho_g}{u_{am}} \frac{\partial e_g}{\partial t} + \phi \frac{\xi \rho_g}{u_{am} + u_g} \frac{\partial u_g}{\partial t} = k_v \frac{\partial^2 u_g}{\partial z^2} + k_h \left( \frac{\partial^2 u_g}{\partial r^2} + \frac{1}{r} \frac{\partial u_g}{\partial r} \right) + \alpha \left[ 1 - \frac{\xi \rho_g Y}{\rho_s} \right]
$$

(4-13)

For small strain problem, there exists $\phi \approx \phi_0$, where $\phi_0$ is the initial porosity. Furthermore, excess gas pressure in MSW landfill normally is a few kPas. Its variation can be neglected compared with atmospheric pressure $u_{am} (101.3 \text{ kPa})$. Therefore there exist $P_g = P_{g0}$ and $\rho_g = \rho_{g0}$, where $P_{g0} = u_{am} + u_{g0}$, $u_{g0}$ is the initial excess gas pressure (kPa), $\rho_{g0}$ is initial gas density (kg/m$^3$). Rearranging Equation (4-13) gives

$$
\frac{\xi \rho_g}{u_{am} + u_{g0}} \frac{\partial e_g}{\partial t} + \frac{\xi \rho_{g0}}{u_{am} + u_{g0}} \phi_0 \frac{\partial u_g}{\partial t} = k_v \frac{\partial^2 u_g}{\partial z^2} + k_h \left( \frac{\partial^2 u_g}{\partial r^2} + \frac{1}{r} \frac{\partial u_g}{\partial r} \right) + \alpha \left[ 1 - \frac{\xi \rho_{g0} Y}{\rho_s} \right]
$$

(4-14)

Incorporating K–H rheological constitutive law of solid matrix (Equation (4-10)) into the gas mass balance equation (Equation (4-14)) in Laplace transform domain gives

$$
\frac{\phi_0}{u_{am} + u_{g0}} (\tilde{s} u_g - u_{g0}) + \tilde{m}_c (\rho_0 g z - \tilde{s} u_g) = \frac{1}{\xi \rho_{g0} g} \left[ k_v \frac{\partial^2 \tilde{u}_g}{\partial z^2} + k_h \left( \frac{\partial^2 \tilde{u}_g}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{u}_g}{\partial r} \right) \right] + \left( \frac{1}{\xi \rho_{g0} \rho_s} - \frac{Y}{\rho_s} \right) \sum_{i=1}^{3} G_i A \lambda_i \frac{e^{-\lambda_i g / H}}{s + \lambda_i}
$$

(4-15)

By introducing dimensionless parameters listed in Appendix 4-A into Equation (4-15), the dimensionless form of governing equation of gas flow is obtained

$$
\frac{1}{\xi} \left[ k_v \frac{\partial^2 \tilde{u}_g}{\partial z^2} + \frac{\partial^2 \tilde{u}_g}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{u}_g}{\partial r} \right] + \left( \tilde{m}_c - 1 \right) \tilde{s} u_g - \tilde{m}_c g \tilde{z} + \left[ \frac{1}{\xi} - \frac{Y \rho_{g0}}{\rho_s} \right] \sum_{i=1}^{3} \frac{A \lambda_i}{s + \lambda_i} e^{-\lambda_i g / H} + \tilde{u}_{g0} = 0
$$
4.3 Solution formulation

Equation (4-16) will be used to find the gas pressure distribution within a deformable landfill with a fully penetrating vertical well connected to some pumping installation. Solution requires specifying boundary conditions. They are given below in dimensionless form:

a) At the landfill base (Figure 4-1(a)):

$$\frac{\partial \tilde{u}_g^*}{\partial z^*} = -\tilde{J}_b^* \text{ at } z^* = 1, \quad r_w^* \leq r^* \leq R_c^*$$  \hspace{1cm} (4-17a)

where $\tilde{J}_b^*$ is Laplace transform of the dimensionless form of the gas flux rate $J_b$ (kg/m$^2$/s) at the base.

b) At the landfill cover, gas flux is controlled by the cover thickness $d_c$ (m) and gas permeability $k_l$ (m/s). Within the landfill cover, gas flow is predominantly vertical (Young 1989; Arigala et al., 1995) and thus the boundary condition at landfill cover is expressed as

$$\frac{\partial \tilde{u}_g^*}{\partial z^*} = L_c^* \tilde{u}_g^* \text{ at } z^* = 0, \quad r_w^* \leq r^* \leq R_c^*$$  \hspace{1cm} (4-17b)

where $L_c^*$ is dimensionless cover property parameter defined as $(k/\mu)l/(d_k k_c)$.

c) The boundary at $r=R_c$ is impermeable, that is

$$\frac{\partial \tilde{u}_g^*}{\partial r^*} = 0 \text{ at } r^* = R_c^*, \quad 0 \leq z^* \leq 1$$  \hspace{1cm} (4-17c)

d) A negative well pressure, $u_w$, is imposed in the well:

$$\tilde{u}_g^* = \tilde{u}_w^*(s) \text{ at } r^* = r_w^*, \quad 0 \leq z^* \leq 1$$  \hspace{1cm} (4-17d)

The initial excess gas pressure throughout the landfill is
Using the analytical method introduced by Chen and Ledesma (2007), the solution of the excessive gas pressure at any time and at any position is expressed in Laplace transform domain as (Appendix 4-B):

\[
\tilde{u}_g^* = \frac{X_n}{K_1(q_n R^*)J_0(q_n r^*) + K_0(q_n r^*)I_1(q_n R^*)} \left[ \cos(\beta_n z^*) + \frac{L_n^*}{\beta_n} \sin(\beta_n z^*) \right] - \sum_{i=1}^{3} \frac{S_i}{T_{bi}} \exp\left(-\lambda_i t^* z^* \right) - \frac{u_0^*}{1 - m_k^a s} + \frac{u_0^*}{1 - m_k^a s}
\]

The definitions of variables in Equation (4-18) are listed in Appendix 4-A. \(I_0, I_1, K_0\) and \(K_1\) are modified Bessel functions of order 0 and 1 of the first and second kind, respectively. The influence of \(u_w\) on gas pressure is reflected by a single term \(X_n (r_w^*, s)\) which is linearly dependent on the vacuum imposed as presented in Equation (4-B.10c). The solution is given in Laplace transform domain and Crump’s method (1976) is used in this paper to perform the numerical inversion of Laplace transform in order to get the solution in real time domain.

Dimensionless gas flux at the well exit, \(Q_{\text{well}}^*\) and through the top cover, \(J_{\text{top}}^*\) are obtained easily by

\[
\tilde{Q}_{\text{well}}^* = \frac{2\pi}{r_w^*} \left[ \frac{\partial \tilde{u}_g^*}{\partial r} \right]_{r_w^*} dz^*
\]

\[
\tilde{J}_{\text{top}}^* = \frac{\partial \tilde{u}_g^*}{\partial z} \bigg|_{z^*=0}
\]

Vertical settlement at any position \((r^*, z^*)\) and at any time within the landfill is obtained by integration as

\[
\tilde{S}^*(r^*, z^*, s) = \int_{z^*} \tilde{E}_r(r^*, z^*, s) dz^*
\]

where \(\tilde{E}_r(r^*, z^*, s)\) is the volumetric strain of waste at position \((r^*, z^*)\) in Laplace transform domain.
\[
\tilde{\varepsilon}_v (r^*, z^*, s) = \phi_b \tilde{m}_{k}^* \left( \frac{\gamma^*}{s} - \tilde{u}_g^* \right)
\]  

(4-22)

Solutions of gas fluxes in the well and across the top cover, as well as settlements within the landfill are also expressed in Laplace transform domain as listed in Appendix 4-B.

### 4.4 Model testing

In spite of the simplifications made in deriving the model, the analytical solution is still quite complex and need testing. Therefore, the new solution is verified by comparison with results from numerical methods. Furthermore, the applicability of the proposed model is verified through comparison with field measurements of gas flux and settlement at two MSW landfills.

#### 4.4.1 Verification of the analytical solution

Solutions of excess gas pressure and settlement are compared with numerical results obtained with CODE_BRIGHT for the dry case \((S_f=0)\). CODE_BRIGHT is a well established finite element code developed for calculating displacements, liquid pressure, gas pressure, temperature and salt content for boundary value problems in saturated or unsaturated soil (Olivella et al., 1996). The code has been modified to incorporate gas generation and porosity change due to degradation of solids. The numerical simulation was conducted in a cylindrical domain as described in Figure 4-1(a) with a height of 20 m, an outer radius of 50 m, and a well radius of 0.5 m. All the parameters used in the numerical simulation are listed in Table 4-1. They are within the typical ranges suggested from previous publications. Geometry of the domain was meshed with 616 quadrilateral elements and 717 nodes as shown in Figure 4-2(a).
Figure 4-2: (a) Mesh and boundary conditions in the numerical simulation. (b) Comparison between numerical simulation and analytical results ($t^* = (k, RT)/(\phi_0, MgH^2)$, $u_0^* = u_0^* (u_{atm} + u_{g0})$, dimensional values with the parameters of Table 4-1 are in parenthesis).
Table 4-1: Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well radius (m)</td>
<td>0.5</td>
</tr>
<tr>
<td>Landfill thickness (m)</td>
<td>20</td>
</tr>
<tr>
<td>Outer radius (m)</td>
<td>50</td>
</tr>
<tr>
<td>Time for filling the landfill (years)</td>
<td>10</td>
</tr>
<tr>
<td>Well pressure (kPa)</td>
<td>-2.0</td>
</tr>
<tr>
<td>Gas flux from the base (a) (kg/m(^2)/s)</td>
<td>-5×10(^{-6})</td>
</tr>
<tr>
<td>Bulk density of refuse (b) (kg/m(^3))</td>
<td>800</td>
</tr>
<tr>
<td>Liquid saturation</td>
<td>0.5</td>
</tr>
<tr>
<td>Porosity of the waste (b)</td>
<td>0.5</td>
</tr>
<tr>
<td>Density of the waste (b) (kg/m(^3))</td>
<td>1100</td>
</tr>
<tr>
<td>Initial excess gas pressure (kPa)</td>
<td>0.0</td>
</tr>
<tr>
<td>Vertical gas conductivity of refuse (c) (m/s)</td>
<td>8.6×10(^{-7})</td>
</tr>
<tr>
<td>Horizontal gas conductivity of refuse (d) (m/s)</td>
<td>2.58×10(^{-6})</td>
</tr>
<tr>
<td>Gas conductivity of the liner (m/s)</td>
<td>8.6×10(^{8})</td>
</tr>
<tr>
<td>Thickness of the liner (m)</td>
<td>0.5</td>
</tr>
<tr>
<td>Total potential of gas generation of the refuse (e) (kg/m(^3))</td>
<td>220</td>
</tr>
<tr>
<td>“Apparent” Henry’s constant (f) (kPa·m(^3)/kg)</td>
<td>180</td>
</tr>
<tr>
<td>Refuse composition (g)</td>
<td></td>
</tr>
<tr>
<td>Readily biodegradable (%)</td>
<td>15</td>
</tr>
<tr>
<td>Moderately biodegradable (%)</td>
<td>55</td>
</tr>
<tr>
<td>Slowly biodegradable (%)</td>
<td>30</td>
</tr>
<tr>
<td>Degradation rate constant (g)</td>
<td></td>
</tr>
<tr>
<td>Readily biodegradable (year(^{-1}))</td>
<td>0.1386</td>
</tr>
<tr>
<td>Moderately biodegradable (year(^{-1}))</td>
<td>0.0231</td>
</tr>
<tr>
<td>Slowly biodegradable (year(^{-1}))</td>
<td>0.017328</td>
</tr>
<tr>
<td>Primary modulus of solid skeleton (kPa)</td>
<td>5000</td>
</tr>
<tr>
<td>Viscosity of solid skeleton (h) (kPa·s)</td>
<td>2.85×10(^{11})</td>
</tr>
<tr>
<td>Residual modulus of solid skeleton (kPa)</td>
<td>350</td>
</tr>
</tbody>
</table>

\(a\) The rate of influx of gas from the base is equivalent to the rate generated by a landfill of 10 m height based on gas generation rate of 4.4×10\(^{-7}\) kg/m\(^3\)/s (Townsend et al., 2005).

\(b\) Value is within the range mentioned by Oweis et al. (1990).

\(c\) Vertical gas conductivity is based on intrinsic permeability of 10\(^{-12}\) m\(^2\) and gas viscosity of 1.37×10\(^{-8}\) kPa·s mentioned by Townsend et al. (2005).

\(d\) The anisotropic ratio is \(k_h/k_v=3.0\).

\(e\) Value is within the range calculated based on the data provided by Merry et al. (2006).

\(f\) The value of \(K_{HG}\) is obtained from Eq.(5b) for equimolar gas mixture of CO\(_2\) and CH\(_4\) at temperature of 310K (\(K_{HG,CO_2}=91.3\) and \(K_{HG,CH_4}=5562.5\)).

\(g\) From Arigala el al. (1995).

\(h\) \(\eta\) is calculated as \(E/\lambda\) where \(\lambda = \sum_{i=1}^{3} A_i \lambda_i = 0.0387\) year\(^{-1}\).
Comparisons between solution given in this paper and numerical simulations by CODE_BRIGHT at two positions are shown in Figure 4-2(b) for both vertical strain and excess gas pressure. As shown in the figure, the results given by two methods are very close to each other which prove the correctness of the given solution.

The analytical solution is much faster and convenient than the numerical method in that the solution can be found quickly at any point, at any time within the region. This feature appears to be especially advantageous for preliminary design studies when geometry of the domain is adjusted repeatedly to find optimal values.

### 4.4.2 Comparison with field measurements of gas flux

The flow from a gas extraction well (A2-II), together with its composition was measured by Martín et al. (2001) at the La Zoreda landfill (Asturias, Spain). The landfill received domestic waste since 1986 with a surface of approximately 700,000 m$^2$. During measurements, the pressure applied in the well varied progressively with a 15-minute waiting period between each change to allow stabilization of gas flow. Entry of air through the cover was deduced from the oxygen contained in the recovered landfill gas. The measured gas flux at the well and infiltrated air are plotted in Figure 4-3 for several well pressures.

Model simulations were performed using the steady-state analytical solution given in Appendix 4-C, with the parameters listed in Table 4-2. The depth of the landfill is 24 m, and the test well penetrates to a depth of 6 m. The waste age in the lower 18 m varies from 2 to 4 years and the average gas production rate is $2.33 \times 10^{-7}$ kg/m$^3$/s (Martín et al., 2001). Gas generated within the lower part of landfill was considered as gas influx $J_b$. In the vicinity of the well, an area of 6 m of the surface was made impermeable. Therefore, simulated infiltrated air was obtained by subtracting the influx within this area from the total influx.

The simulated gas flux in Figure 4-3 is quite similar to that measured in the field and exhibits a linear relationship with applied pressure. This is consistent with Equation (4-C.2a) which indicates a quasi-linear relationship between gas flux at the well, $Q_{well}$ and well pressure, $u_w$. The simulated results show that for $u_w > -0.2$ kPa, the area where air enters the landfill is controlled within the area of 6 m. This is exactly consistent with what measured in the test, that is, no oxygen was measured in the composition of recovered gas under a small vacuum.
4.4.3 Comparison with field measurements of settlement

A total of 29 settlement monuments, with brass tag set in concrete, were monitored from 1964 to 1981 at the Mission Canyon landfill (Los Angeles, CA). The landfill received commercial and residential refuse between 1960 and 1964. Measurements of monument 113 reported by Coduto and Huitric (1990) were used for comparison with model simulation. Due to the lack of information about the landfill, the parameters used in the simulation are based on the general conditions of MSW landfills (Table 4-3).
The analytical solution for settlement in 1-D problem was obtained by coupling mechanical compression described by K–H rheological model and gas flow based on Darcy’s law (Yu et al., 2010). The landfill is treated as impermeable at the base and free venting at the top surface. The modulus of solid was adjusted to best fit the field measurements. The comparison between model simulation and field measurements is presented in Figure 4- 4. The excellent agreement proves the capability of the proposed model in this paper in reproducing the evolution of settlement for MSW landfills.

<table>
<thead>
<tr>
<th>Landfill depth (m)</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity of the waste</td>
<td>0.5</td>
</tr>
<tr>
<td>Bulk density of waste $^a$ (kg/m$^3$)</td>
<td>760</td>
</tr>
<tr>
<td>Density of solid phase (kg/m$^3$)</td>
<td>1500</td>
</tr>
<tr>
<td>Gas generation potential $^a$ (kg/m$^3$)</td>
<td>157</td>
</tr>
<tr>
<td>Reaction-rate constant of the waste $^b$ (s$^{-1}$)</td>
<td>$3.67 \times 10^{-9}$</td>
</tr>
<tr>
<td>Gas conductivity of refuse $^c$ (m/s)</td>
<td>$8.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>Primary modulus of solid skeleton (kPa)</td>
<td>5000</td>
</tr>
<tr>
<td>Residual modulus of solid skeleton (kPa)</td>
<td>440</td>
</tr>
<tr>
<td>Thickness of the liner (m)</td>
<td>1.0</td>
</tr>
<tr>
<td>Gas conductivity of the liner (m/s)</td>
<td>$8.6 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

$^a$ Value is measured by Nastev et al. (2001).
$^b$ Referring to Liu et al. (2006).
$^c$ Based on intrinsic permeability of $10^{-12}$ m$^2$ and gas viscosity of $1.37 \times 10^{-8}$ kPa·s mentioned by Townsend et al. (2005).

Figure 4- 4: Comparison of field measurement and model simulation of settlement at Mission Canyon landfill at Mission Canyon landfill.
4.5 Applications

The influences of various material properties and operation strategies on the optimal determination of operational well pressure and well distance are investigated in this section to gain insight into the physics of the coupled system. Two concepts are used here to illustrate the collection efficiency of the extraction well, gas recovery ratio and radius of influence. Gas recovery ratio is defined as the ratio of LFG flow rate at the well exit, divided by the total gas generation rate in a cylinder volume bounded by outer radius $R_e$. Radius of influence is defined as the distance from the centre of extraction well which bounds a cylinder volume within which gas recovery ratio is 90%. The concept of radius of influence is helpful in the determination of optimum well distance which can be designed approximately as twice of radius of influence.

4.5.1 Effects of variation of mass storage of gas

Variation of mass storage of gas is caused by the deformation of solid matrix, the degradation of solids, and the depressurization of gas which causes reduction in gas density and degassing of dissolved gas components. The effect of these processes is shown in Figure 4-5, which displays excess gas pressure at $r=10$ m, $z=10$ m.

It is clear that compressibility and dissolution effects are only relevant for early time response. That is, they are important for pump test interpretation or for intermittent extraction designs. However, they are negligible for long-term continuous pumping because so are $\frac{\partial u_g}{\partial t}$ and $\frac{\partial \epsilon_v}{\partial t}$. Under these conditions, the problem becomes quasi-steady-state. That is, the problem can be approximated as steady-state with an internal source, $\alpha$, that decays slowly with time. Therefore, $u_g$ obeys

$$\frac{k_v}{k_h} \frac{\partial^2 u_g^*}{\partial z^2} + \frac{\partial^2 u_g^*}{\partial r^2} + \frac{1}{r} \frac{\partial u_g^*}{\partial r} + \alpha^* = 0 \quad (4-23)$$

All the dimensionless variables are defined as in Appendix 4-A and 4-C. In a sufficiently mature landfill in which $\alpha$ is assumed to be constant and the variation rate of mass storage of gas is negligible. Under this circumstance, analytical solutions for the excess gas pressures, gas fluxes in the well and across the top cover are given in Appendix 4-C with the same boundary conditions as those listed in Equations (4-17a)-(4-17d). The governing equation of steady-state problem reduces dramatically the number of input parameters and has great advantage in the preliminary design of optimum LFG control systems.
Chapter 4 — Gas flow to a vertical gas extraction well in deformable MSW landfills

### 4.5.2 Effects of operational vacuum

Appropriate well distance for a certain vacuum applied in the extraction well is an important parameter in the design of LFG control systems. The effect of well pressure on the distribution of excess gas pressure around the well is plotted in Figure 4-6. Except well pressure, all other parameters are the same as those listed in Table 4-1. Horizontal gas pressure contour lines parallel to the top cover indicate the area where gas escapes through the cover, while steep gas pressure contour lines beside the extraction well indicate the area where gas flows towards the extraction well.

Flow patterns change significantly when operational vacuum changes. The size of the well capture zone increases with the imposed vacuum which implies an increased recovery at the well. Still, increases in the gas flux at the well are caused both by increases in the radius of influence (i.e., increase in the amount of generated gas captured by the extraction well) and by increases in the top cover area under suction (i.e., increase in the proportion of air sucked by the well). Both factors are discussed below.

The variation of gas flux across the top cover $J_{top}$ with radial distance $r$ is plotted in dimensionless form for several vacuum imposed (Figure 4-7). Atmospheric air enters the landfill in the vicinity of the pumping well. A part of the infiltrated air flows outwards through the extraction well, the rest is retained and consumed in the waste and contributes to the increase of CO$_2$ and N$_2$ concentrations in the landfill gas. This is undesirable not only...
because of the energy wasted in pumping air but also because oxygen concentration inhibits anaerobic biodegradation. The higher the vacuum is imposed in the well, the larger is the area through which atmospheric air enters the landfill.

Figure 4- 6: Distribution of excess gas pressure (kPa) under different well pressure, 5 years after closure (a) \( u_w = 0.0 \text{ kPa} \) (b) \( u_w = -1.5 \text{ kPa} \) (c) \( u_w = -3 \text{ kPa} \) (d) \( u_w = -4.5 \text{ kPa} \) (\( r^* = r/H, z^* = z/H \)).

Figure 4- 7: Gas fluxes across the top cover with different well pressure, 5 years after closure (negative flux means that gas emits into ambient) (\( r^* = r/H, J_{top}^* = (gH/k_v/(u_{atm}+u_g)) J_{top} \)).
Considering that a portion of the extracted gas may be atmospheric air sucked into the landfill by the extraction itself, LFG flow rate at the well exit is assumed to be the subtraction of influx of air through top cover from total gas flow rate at the well exit. This approximation underestimates the efficiency of the extraction well. For the normal case listed in Table 4-1, infiltrated air is only 5% of the total gas flux in the well, and there will not be much error for such an approximation. The error will increase with vacuum imposed in the well and the permeability of the top cover. Radius of influence can be determined according to the curves shown in Figure 4-8. Setting variable outer radius \( R_e \) is actually equivalent to changing the distance between two neighbouring wells. The results apparently show that the gas recovery ratio decreases with \( R_e \) which implies that well efficiency of capturing gas decreases with well distance. The reason is obvious, because the ability of the well to capture the far-away landfill gas decays with the increase of the radial distance from the well. For the case of a passive well (vacuum is equal to zero), about 40% of the gas produced by the waste emits out into the ambient through the landfill surface when the half well distance, \( R_e \), is equal to the depth of the landfill.

Radius of influence increases with the vacuum imposed at the extraction well. It increases from 0.5 \( H \) to 2 \( H \) (\( H \) is the depth of the landfill) as the well pressure varies from zero to -4.5 kPa.

![Figure 4-8: Gas recovery ratio as a function of half distance between wells for different well pressures, 5 years after closure (\( R_e^* = Re/H \)).](image-url)
4.5.3 Effects of permeability anisotropic ratio

Behaviour of gas flow around extraction well is much influenced by gas permeability of the refuse. Horizontal permeability \( k_h \) is usually greater than vertical permeability \( k_v \) for MSW landfills because of both the layered deposition process and extensively used daily cover during waste deposition.

Based on the previous definition of gas recovery ratio and radius of influence, gas recovery ratio versus \( R_e \) for different permeability anisotropic ratios (defined as \( k_h / k_v \)) in the case of \( u_w = -2 \text{kPa} \) is plotted in Figure 4-9. Radius of influence increases with anisotropic ratio of the waste for fixed \( k_v \), and varies from 21 m to 38 m as the ratio of \( k_h / k_v \) increases from 1 to 10. The reason is evident because gas is prone to flow horizontally towards the extraction well at higher permeability anisotropic ratio.

![Figure 4-9: Gas recovery ratio as a function of half distance between wells for different anisotropic ratios, 5 years after closure (\( R_{e^*} = R_e/H \)).](image)

4.5.4 Effects of cover properties

Impact of cover properties on the gas flow pattern can be studied by the dimensionless parameter \( L_c^* \) which is defined as \( (k_f H)/(d_1 k_v) \). The influence of \( L_c^* \) on gas flow around the extraction well is examined with all the input data shown in Table 1 except the ratio of \( k_f \) and \( d_1 \).
Influx of the atmospheric air through the cover is clearly demonstrated in Figure 4-10 which shows the variation of gas flux across the top cover with radial distance \( r \) under well pressure equal to -2 kPa. Atmospheric air enters the landfill in the vicinity of the well and gas flows outwards through the cover in the larger radius. Higher value of \( L_c^* \) (thinner and more permeable cover) will cause larger area where atmospheric air enters the landfill.

Variation of gas recovery ratio with \( L_c^* \) under different pressure applied in the extraction well, \( u_w \), is shown in Figure 4-11. Gas recovery ratio decreases with \( L_c^* \) which implies that more gas flows outwards into the ambient through the top cover when the cover is thinner and more permeable. Furthermore, gas recovery ratio is not so much influenced by the permeability of the top cover for an active well imposed with high vacuum as for a passive venting well. That means with higher vacuum imposed in the extraction well, increasing the thickness of the top cover is not so efficient to enhance the gas recover ratio as with lower vacuum.

\[
J_{\text{top}}^* = \left( \frac{gH}{k_v} \right) \left( \frac{(u_{\text{atm}} + u_{\text{g0}})}{H} \right) J_{\text{top}}.
\]

Figure 4-10: Variation of gas flux across the top cover with distance for different cover properties, 5 years after closure (negative flux means flowing out of the landfill) \( (r^* = r/H, L_c^* = k_iH/(d_i k_v), J_{\text{top}}^* = \left( gH/k_v (u_{\text{atm}} + u_{\text{g0}}) \right) J_{\text{top}}) \).
4.5.5 Suggestions to optimum design of LFG control systems

Optimal well distances (twice the radius of influence) under various combinations of applied excess gas pressure in the extraction well, \( u_w \) and cover properties, \( L_c^* \) are shown in Figure 4-12. Gas generation rate is equivalent to \( 3.87 \times 10^{-7} \) kg/m\(^3\)/s 5 years after closure. The parameters in the base case with \( L_c^* = 4 \) are the same as those in Table 1. Smaller \( L_c^* \) (thicker and less permeable top cover) and lower \( u_w \) (higher vacuum applied in the well) give greater optimal well distance.

Figure 4-12 implies a quasi-linear relationship between optimal well distance and \( u_w \) for the case of \( L_c^* = 0.04 \), when very little LFG or atmospheric air flow across the top cover. When the landfill is covered with a thinner and more permeable top cover (larger \( L_c^* \)), the optimal well distance no longer increases proportionally to \( u_w \). The increase in the optimal well distance per unit change in \( u_w \) is more significant for small vacuum imposed. For example, for the base case, the optimal well distance increases from 20 m to 60 m when \( u_w \) varies from 0 kPa to -2 kPa, while it only increases from 60 m to 80 m when \( u_w \) varies from -2 kPa to -4.5 kPa. High vacuum yields larger gas recovery in the extraction well but also induces more undesirable air influxes. The portion of infiltrated air contained in the recovered gas reduces the efficiency in recovering biogas. Therefore, solely increasing the vacuum
imposed at the extraction well will not efficiently lead to larger optimal well distance, especially when the applied vacuum is high or the top cover is thinner and more permeable.

Large optimal well distances can be obtained by decreasing $u_w$ or $L_c^*$. Decreasing $L_c^*$ requires increasing the thickness of the top cover, or decreasing its permeability, or both. For a given well distance, the most economical combination of $u_w$ and $L_c^*$ can be deduced by comparing the cost of decreasing $u_w$ with the cost of decreasing $L_c^*$. The curves shown in Figure 4-12 provide an engineering tool to find the most economical strategy of LFG control systems by seeking the balance among well distance, cover properties and vacuum imposed at the extraction well.

![Figure 4-12: Optimal well distances under various combination of well pressures and cover properties, 5 years after closure ($L_c^* = k_l H/(d_k v)$).](image)

**4.6 Summary and conclusions**

An analytical solution to predict gas flow to a vertical gas extraction well has been presented. The solution accounts for the effects of exponential decaying gas generation, mass storage variations due to compression of gas and refuse, dissolution of gas components and porosity enlargement due to solids degradation. Mechanical compression of the refuse is coupled with gas pressure using K-H rheological model. The solution has been obtained analytically in Laplace transform domain for excess gas pressures. Extracted gas fluxes in the well and across the top cover, as well as landfill settlements have also been derived analytically. The solution is applicable to an anisotropic landfill with a low-permeable final
cover, and bounded at some depth by either the water table or an impermeable liner. The solution has been verified by comparison with field measurements and numerical simulation.

Storage terms are only relevant during early time (i.e., up to a few days). Afterwards, gas flow pattern is quasi-steady state solely controlled by gas generation rate. Therefore, the model proposed in this paper can be simplified to be steady-state flow with fixed gas generation rate. This implies that optimum extraction strategy can be obtained with little paid for past extraction. This facilitates real time optimal operation and control. Analytical solutions for excess gas pressures, gas fluxes in the well and across the top cover are also provided for steady state.

Parametric studies indicate that the solution to the proposed model is convenient in the optimization of gas extraction systems, including well distance, cover properties and vacuum imposed in the extraction well. The expression of gas fluxes in the well, $Q_{\text{well}}$, shows that there exists a quasi-linear relationship between vacuum imposed and $Q_{\text{well}}$. The study shows that both the vacuum applied in the well and anisotropic ratio of refuse will clearly improve the well efficiency in recovering biogas. Radius of influence increases from 0.5 $H$ to 2 $H$ as well pressure varies from zero to -4.5 kPa, and increases from 0.92 $H$ to 2.6 $H$ as the permeability anisotropic ratio varies from 1 to 10. The phenomenon that more atmospheric air enters into the landfill in the vicinity of the active extraction well through the top cover indicates that increasing the vacuum imposed in the extraction well will not result larger radius of influence proportionally. An economical balance should be carefully sought among the vacuum imposed, well distance and properties of the top.

The proposed model is a promising engineering tool for preliminary approximations and guidance in the design of LFG control systems. Despite the fact that the model is established based on some simplifications, it is capable of describing quite accurate settlement evolutions, and gas flow patterns within MSW landfills.
Appendix 4-A: Definition of dimensionless parameters

The dimensionless variables used in the model are defined as follows:

\[
\begin{align*}
    z^* &= \frac{z}{H}, & r^* &= \frac{r}{H}, & R_e^* &= \frac{R_e}{H}, & r_w^* &= \frac{r_w}{H}, \\
    t^* &= \frac{k_i RT \phi_0 MgH^2}{\rho_0 MgH^2}, & t_f^* &= \frac{k_i RT \phi_0 MgH^2}{\rho_0 MgH^2}, & \lambda_i^* &= \frac{\phi_i MgH^2}{k_i RT} \lambda_i, & \frac{1}{K_{HG}^*} &= \frac{1}{K_{HG}} RT M (4-A.2)
\end{align*}
\]

\[
\begin{align*}
    u_w^* &= \frac{u_w}{u_{am} + u_{g0}}, & u_g^* &= \frac{u_g}{u_{am} + u_{g0}}, & u^* &= \frac{u_g}{u_{am} + u_{g0}} (4-A.3)
\end{align*}
\]

\[
\begin{align*}
    G_r^* &= \frac{G_r}{\rho_{g0} \phi_0}, & \gamma^* &= \frac{H \rho_{g0} g}{u_{am} + u_{g0}}, & J_b^* &= \frac{g}{k_v \left( u_{am} + u_{g0} \right)} H, & L_c^* &= \frac{k_i H}{d_l k_v} (4-A.4)
\end{align*}
\]

\[
\begin{align*}
    E^* &= \frac{E}{u_{am} + u_{g0}}, & E_0^* &= \frac{E_0}{u_{am} + u_{g0}} (4-A.5)
\end{align*}
\]

\[
\begin{align*}
    \eta^* &= \frac{k_i RT \phi_0 MgH^2}{\left( u_{am} + u_{g0} \right)} \eta, & \tilde{m}_k^* &= -\frac{1}{\phi_0} \left( \frac{1}{E^* + \eta^*} + \frac{1}{E_0^*} \right) (4-A.6)
\end{align*}
\]

\[
\begin{align*}
    Q_{well}^* &= \frac{g Q_{well}}{k_h \left( u_{am} + u_{g0} \right)}, & J_{top}^* &= \frac{g H J_{top}}{k_v \left( u_{am} + u_{g0} \right)} (4-A.7)
\end{align*}
\]

Appendix 4-B: Development of solutions

To solve non-homogeneous differential equation Equation (4-16), it is supposed that the general solution \( \tilde{u}^* \left( r^*, z^*, s \right) \) is the sum of the homogenous solution \( v \left( r^*, z^*, s \right) \) and one particular solution \( w \left( z^*, s \right) \) which is chosen to satisfy the following differential equation...
\[
\frac{k_v}{k_h} \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \xi \left(1 - \tilde{m}_k^*\right)s - \xi \tilde{m}_k^* \gamma^* z^* + \sum_{i=1}^{3} S_i \exp\left(-\lambda_i^* t_f^* z^*\right) + u_{y0}^* = 0 \quad (4-B.1a)
\]

and boundary conditions:
\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial w(0,s)}{\partial z^*} = L_i w(0,s) \\
\frac{\partial w(1,s)}{\partial z^*} = -J_b^* (s)
\end{array} \right. \\
(4-B.1b)
\]

where \( S_i = \left(1 - \frac{\xi \rho_{r0}^* Y}{\rho_s^*}\right) G_r^* \frac{A_i^* \lambda_i^*}{s + \lambda_i^*} \).

The solution \( w(z^*, s) \) is immediately determined from Equations (4-B.1a) and (4-B.1b) to be
\[
w(z^*, s) = C_1 \exp(T_a z^*) + C_2 \exp(-T_a z^*) + \sum_{i=1}^{3} \frac{S_i}{T_{bi}} \exp\left(-\lambda_i^* t_f^* z^*\right) - \frac{\tilde{m}_k^* \gamma^* z^*}{\left(1 - \tilde{m}_k^*\right)s} + \frac{u_{y0}^*}{\left(1 - \tilde{m}_k^*\right)s} \quad (4-B.2a)
\]

where \( T_a = \sqrt{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v} \), \( T_{bi} = \xi \left(1 - \tilde{m}_k^*\right)s k_r^2 / k_h \),
\[
\begin{align*}
C_1 &= \left[ \frac{\sum_{i=1}^{3} \frac{S_i}{T_{bi}} \left(\lambda_i^* t_f^* + L_i^*\right) + \frac{\tilde{m}_k^* \gamma^* + L_i^* u_{y0}^*}{\left(1 - \tilde{m}_k^*\right)s}}{1 - \left(-2 T_a\right) - \frac{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v}{1 - \tilde{m}_k^*}} \right] \exp\left(-2 T_a\right) + \frac{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v}{1 - \tilde{m}_k^*} - \frac{\sum_{i=1}^{3} \frac{S_i}{T_{bi}} \left[\lambda_i^* t_f^* \exp\left(\lambda_i^* t_f^*\right)\right]}{1 - \left(-2 T_a\right) - \frac{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v}{1 - \tilde{m}_k^*}} \exp\left(-2 T_a\right) \] \\
C_2 &= \left[ \frac{\sum_{i=1}^{3} \frac{S_i}{T_{bi}} \left(\lambda_i^* t_f^* + L_i^*\right) + \frac{\tilde{m}_k^* \gamma^* + L_i^* u_{y0}^*}{\left(1 - \tilde{m}_k^*\right)s}}{1 - \left(-2 T_a\right) - \frac{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v}{1 - \tilde{m}_k^*}} \right] \exp\left(-2 T_a\right) - \frac{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v}{1 - \tilde{m}_k^*} - \frac{\sum_{i=1}^{3} \frac{S_i}{T_{bi}} \left[\lambda_i^* t_f^* \exp\left(\lambda_i^* t_f^*\right)\right]}{1 - \left(-2 T_a\right) - \frac{\xi \left(1 - \tilde{m}_k^*\right)s k_h / k_v}{1 - \tilde{m}_k^*}} \exp\left(-2 T_a\right) \] \\
(4-B.2b)
\]

The homogeneous solution \( v(r^*, z^*, s) \) satisfies the following homogeneous differential equation
\[
\xi \left(1 - \tilde{m}_k^*\right)s v = \frac{k_v}{k_h} \frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} \quad (4-B.3a)
\]

and the following homogeneous boundary conditions:
\[
\begin{align*}
\frac{\partial v(r^*, 0, s)}{\partial z^*} &= L^*_v v(r^*, 0, s) \\
\frac{\partial v(r^*, 1, s)}{\partial z^*} &= 0
\end{align*}
\] (4-B.3b)

\[
\begin{align*}
\frac{\partial v(R^*_w, z^*, s)}{\partial r^*} &= 0 \\
v(r^*_w, z^*, s) &= \tilde{u}_w(s) - w(z^*, s)
\end{align*}
\] (4-B.3c)

Now \(v(r^*, z^*, s)\) is supposed to be the product of two new variants \(X(r^*, s)\) and \(Y(z^*, s)\)

\[
v(r^*, z^*, s) = X(r^*, s) \cdot Y(z^*, s)
\] (4-B.4)

Substituting Equation (4-B.4) into Equation (4-B.3a) yields two partial differential equations concerning \(X(r^*, s)\) and \(Y(z^*, s)\) respectively:

\[
Y''(z^*, s) + \chi Y(z^*, s) = 0
\] (4-B.5a)

with associated boundary conditions:

\[
Y'(0, s) = L'_v Y(0, s)
\] (4-B.5b)

\[
Y'(1, s) = 0
\] (4-B.5c)

\[
\left[1 - \tilde{m}_k^u\right] s + \chi(k, / k_h) X(r^*, s) = X'(r^*, s) - X(r^*, s) / r^* = 0
\] (4-B.6a)

with associated boundary conditions

\[
X'(R^*_w, s) = 0
\] (4-B.6b)

\[
X'(r^*_w, s) = \tilde{u}_w(s) - w(z^*, s)
\] (4-B.6c)

where \(\chi\) = separation constant.

(1) when \(\chi < 0\), \(Y(z^*, s) = A \exp(\sqrt{-\chi} z^*) + B \exp(-\sqrt{-\chi} z^*)\)

\[
B = A \exp(2\sqrt{-\chi} H^*).
\]
Under the confining conditions of \( c_L > 0, \chi \neq 0 \), the boundary condition (B.5b) determines that there is non-zero solutions of \( Y(z^*, s) \).

(2) When \( \chi = 0 \), the general solution to Equation (4-B.5a) is \( Y(z^*, s) = Az^* + B \). Boundary condition Equation (4-B.5c) yields \( A = 0 \), Equation (4-B.5c) requires \( A \equiv B \equiv 0 \). There is non-zero solutions of \( Y(z^*, s) \) when \( \chi = 0 \).

(3) When \( \chi > 0 \), the non-zero solution of \( Y(z^*, s) \) is

\[
Y_n(z^*, s) = A_n \left[ \cos(\beta_n z^*) + \frac{L'_c}{\beta_n} \sin(\beta_n z^*) \right]
\]

(4-B.7a)

where \( A_n \) is constant needed to be determined; \( \chi_n = \beta_n^2 (\beta_n > 0) \); \( \beta_n \) are the solutions of

\[
\tan(\beta_n) = \frac{L'_c}{\beta_n}, n=1,2,3,\ldots.
\]

(4-B.7b)

The solution of \( X(r^*, s) \) according to the partial differential equation Equation (4-B.6a) takes the form

\[
X_n(r^*, s) = D_n I_0(q_n r^*) + B_n K_0(q_n r^*)
\]

(4-B.8)

where \( D_n \) and \( B_n \) are constants needed to be determined; \( I_0 \), \( K_0 \) are modified Bessel functions of the first and second kind of zero order respectively; \( q_n^2 = D_g \left[ 1 - \tilde{m}_x^a \right] s + \beta_n^2 k_r / k_h, n=1,2,3,\ldots. \)

Substituting Equations (4-B.7a) and (4-B.8) into Equation (4-B.4), the solution \( v(r^*, z^*, s) \) can be expressed in Fourier series expansion form as

\[
v_n(r^*, z^*, s) = \sum_{n=1}^{\infty} \left[ C_{1n} I_0(q_n r^*) + C_{2n} K_0(q_n r^*) \right] \left[ \cos(\beta_n z^*) + \sin(\beta_n z^*) \frac{L'_c}{\beta_n} \right]
\]

(4-B.9)

where \( C_{1n} \) and \( C_{2n} \) are constants needed to be determined according to the boundary condition (Equation (4-B.3c)). Based on the orthogonality associated eigenfunctions, the constants \( C_{1n} \) and \( C_{2n} \) are found to be

\[
C_{1n} = \frac{X_n(r_w^*, s) K_0(q_n R_c^*)}{I_0(q_n r_w^*) + K_0(q_n r_w^*) I_0(q_n R_c^*)}
\]

(4-B.10a)
\[
C_{2n} = \frac{X_{p}\left(r_w^*, s\right)J_l\left(q_n R_e^*\right)}{K_1(q_n R_e^*)J_0(q_n r_w^*) + K_0(q_n r_w^*)J_1(q_n R_e^*)} \\
(4-B.10b)
\]

where

\[
X_{s}\left(r_w^*, s\right) = \frac{2L_s^* u_w^*(s) + \frac{2\beta_s^2}{q_w^*} \left[ \sum_{n=1}^{\infty} \frac{S_n \beta_n^* t_s^* \exp(-\lambda_n^* t_s^*)}{\cos(\beta_n^* \beta_n^* t_s^* + \beta_s^2)} \right] \frac{2 \gamma_n^* y^* + 2 \gamma_u^* L_s^*}{q_s^*} + \frac{2 k_1 \beta_s^2 J_1(s) s + 2 \gamma_n^* y^*}{\cos(\beta_n^* q_s^*)}}{\beta_n^* + L_s^* + L_s^*} \\
(4-B.10c)
\]

\(I_1, K_1\) are modified Bessel functions of the first and second kind of first order respectively.

Hence, the final solution for the excess gas pressure in the waste is expressed using dimensionless parameters in Laplace transform domain as

\[
\tilde{u}_g^* = \sum_{n=1}^{\infty} \frac{X_n\left(r_w^*, s\right)\left[K_1(q_n R_e^*)J_0(q_n r_w^*) + I_1(q_n R_e^*)K_0(q_n r_w^*)\right]}{K_1(q_n R_e^*)J_0(q_n r_w^*) + K_0(q_n r_w^*)J_1(q_n R_e^*)} \left[ \cos(\beta_n^* z^*) + \frac{L_s^*}{\beta_n^*} \sin(\beta_n^* z^*) \right] + w(z^*, s) \\
(4-B.11)
\]

The vertical displacement at position \((r^*, z^*)\), in the waste is obtained based on the volumetric strain of waste skeleton

\[
\tilde{S}^* (r^*, z^*, s) = \int_{z^*}^{s} \phi_n \tilde{m}_e^* \left( \frac{y^* z^* - \tilde{u}_g^*}{s} \right) dz^* \\
(4-B.12)
\]

Substituting expression of \(\tilde{u}_g^*\) in Equation (4-B.11) into Equation (4-B.12), the final expression of vertical displacement at any position \((r^*, z^*)\) is expressed in dimensionless form in Laplace transform domain as

\[
\tilde{S}^* = \phi_n \tilde{m}_e^* \frac{y^* (1 - z^*)^2}{2s} - \phi_n \tilde{m}_e^* \left[ \frac{C_1}{T_a} \exp(T_a z^*) - \frac{C_2}{T_a} \exp(-T_a z^*) - \sum_{i=1}^{3} \frac{S_i \exp(-\lambda_i^* t_f^*)}{T_a \lambda_i^* t_f^*} - \frac{2 u_{i0}^* - \tilde{m}_e^* y^*}{2(1 - \tilde{m}_e^* z^*)/s} \right] - \left[ \frac{C_1}{T_a} \exp(T_a z^*) - \frac{C_2}{T_a} \exp(-T_a z^*) - \sum_{i=1}^{3} \frac{S_i \exp(-\lambda_i^* t_f^*)}{T_a \lambda_i^* t_f^*} + \frac{2 u_{i0}^* - \tilde{m}_e^* y^* z^*}{2(1 - \tilde{m}_e^* z^*)/s} \right] \\
(4-B.13)
\]

The extracted gas fluxes at the well exit takes the form:
\[ \tilde{Q}^*_\text{well} = -2\pi r^*_w \int_0^\infty \frac{\partial \tilde{u}^*_g}{\partial r} \left|_{r^*_w} \right. \, dz^* = -\frac{2\pi r^*_w L^*_c}{\beta^2} \sum_{n=1}^\infty q_n X_n \left( r^*_w, s \right) \left[ K_1 \left( q_n R^*_c \right) I_0 \left( q_n r^*_w \right) - I_1 \left( q_n R^*_c \right) K_0 \left( q_n r^*_w \right) \right] \left[ K_1 \left( q_n R^*_c \right) I_0 \left( q_n r^*_w \right) + K_0 \left( q_n r^*_w \right) I_1 \left( q_n R^*_c \right) \right] \]

\[ \text{(4-B.14)} \]

Gas fluxes across the top cover (negative means outflow of gas through the cover) is:

\[ \bar{J}^*_\text{top} = \left. \frac{\partial \tilde{u}^*_g}{\partial z^*} \right|_{z^*=0} = -L^*_c \sum_{n=1}^\infty X_n \left( r^*_w, s \right) \left[ K_1 \left( q_n R^*_c \right) I_0 \left( q_n r^*_w \right) + I_1 \left( q_n R^*_c \right) K_0 \left( q_n r^*_w \right) \right] \left[ K_1 \left( q_n R^*_c \right) I_0 \left( q_n r^*_w \right) + K_0 \left( q_n r^*_w \right) I_1 \left( q_n R^*_c \right) \right] \]

\[ -L^*_c \left[ C_1 + C_2 + \sum_{n=1}^3 \left( S_1 / T_{bi} \right) + u^*_g \left( 1 - \bar{m}^*_g \right) / s \right] \]

\[ \text{(4-B.15)} \]
Appendix 4-C: Solutions for steady state condition

The dimensionless form of fixed gas production rate $\alpha$ is $\alpha^* = gH^2\alpha/(u_{am} + u_{e,0})/k_h$.

The excess gas pressure in the waste is expressed using dimensionless parameters as:

$$u_g^*(r^*, z^*) = \sum_{n=1}^{\infty} X_n(r_w^*) \left[ \frac{K_1(q_n R_c^*) I_0(q_n r_v^*) + I_1(q_n R_c^*) K_0(q_n r_v^*)}{K_1(q_n R_c^*) I_0(q_n r_w^*) + K_0(q_n r_w^*) I_1(q_n r_v^*)} \right] \left[ \cos(\beta_n z^*) + \frac{L_c^*}{\beta_n^*} \sin(\beta_n z^*) \right]$$

$$- \alpha^* k_h z^*/(2k_v) + \left( \alpha^* k_h / k_v - J_b^* \right) z^* + 1/L_c^*$$

(4-C.1a)

where: $\beta_n$ are the solutions of $\tan(\beta_n) = L_c^*/\beta_n$, $n = 1, 2, 3, ...$.

(4-C.1b)

$$q_n^2 = \beta_n^* k_v / k_h \quad n = 1, 2, 3, ...$$

(4-C.1c)

$$X_n(r_w^*) = \frac{2L_c^* u_w^* + 2J_b^* \cos \beta_n - 2\alpha^* L_c^* / q_n^2}{\beta_n^* + L_v^2 + L_c^*}$$

(4-C.1d)

Gas fluxes at the well exit takes the form:

$$Q_{well}^* = -2\pi w^* \int_{0}^{\pi} \frac{\partial u_g^*}{\partial r^*} \, dz = F(r_w^*, q_n^*) u_w^* + G(r_w^*, q_n^*)$$

(4-C.2a)

where: $F(r_w^*, q_n^*) = -\sum_{n=1}^{\infty} K_1(q_n r_v^*) I_0(q_n r_w^*) - I_1(q_n r_v^*) K_0(q_n r_w^*) / \left( K_1(q_n r_v^*) I_0(q_n r_w^*) + K_0(q_n r_w^*) I_1(q_n r_v^*) \right) \left[ \beta_n^* + L_v^2 + L_c^* \right] k_h q_n$.

(4-C.2b)

$$G(r_w^*, q_n^*) = \sum_{n=1}^{\infty} \left[ \frac{\alpha^*}{q_n^2} - \frac{J_b^*}{L_v^* \cos(\beta_n)} \right] K_1(q_n R_c^*) I_0(q_n r_v^*) - I_1(q_n R_c^*) K_0(q_n r_v^*) / \left( K_1(q_n r_v^*) I_0(q_n r_w^*) + K_0(q_n r_w^*) I_1(q_n r_v^*) \right) \left[ \beta_n^* + L_v^2 + L_c^* \right] k_h q_n$$

(4-C.2c)

The expression of gas fluxes across the top cover is (negative value means gas flowing out through the cover):

$$J_{top}^* = -L_c^* \sum_{n=1}^{\infty} X_n(r_v^*) \left[ \frac{K_1(q_n R_c^*) I_0(q_n r_v^*) + I_1(q_n R_c^*) K_0(q_n r_v^*)}{K_1(q_n R_c^*) I_0(q_n r_w^*) + K_0(q_n r_w^*) I_1(q_n R_c^*)} \right] \alpha^* k_h / k_v$$

(4-C.3)
CHAPTER 5

A hybrid method for quasi-three-dimensional slope stability analysis in a municipal solid waste landfill

Abstract

Limited space for accommodating the ever increasing mounds of municipal solid waste (MSW) demands the capacity of MSW landfill be maximized by building landfills to greater heights with steeper slopes. This situation raises concerns regarding the stability of high MSW landfills. A hybrid method for quasi-three-dimensional slope stability analysis based on the finite element stress analysis was applied in a case study at a MSW landfill in east-northern Spain. Potential slides can be assumed located within the waste mass due to the lack of weak foundation soils and geosynthetic membranes at the landfill base. The only triggering factor of deep-seated slope failure is the higher leachate level and the relatively high and steep slope in the front. The valley-shaped geometry and layered construction procedure at the site make three-dimensional slope stability analyses necessary for this landfill. In the finite element stress analysis, variations of leachate level during construction and continuous settlement of the landfill were taken into account. The 'equivalent' three-dimensional factor of safety (FoS) was computed from a series of two-dimensional analysis results for evenly spaced cross sections within the potential sliding body. Results indicate that the hybrid method for quasi-three-dimensional slope stability analysis adopted in this paper is capable of

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locating roughly the spatial position of the potential slide mass. This easily manipulating method can serve as an engineering tool in the preliminary estimate of the FoS as well as the approximate position and extent of the potential slide mass. The result that FoS obtained from three-dimensional analysis increases as much as 50% comparing to that from two-dimensional analysis implies the significance of the three-dimensional effect for the case landfill. Influences of time elapsed after landfill closure, leachate level as well as unit weight of waste on FoS were also investigated in this paper. These sensitivity analyses help the planning of construction practices and operating procedures in the case landfill.

Keywords: municipal solid waste; slope stability; finite element stress analysis; limit equilibrium; hybrid method; quasi-three-dimensional analysis

5.1 Introduction

Shortage of space for accommodating the increasing mounds of waste poses a challenge to maximize the capacity of the municipal solid waste (MSW) landfill by increasing the height and slope inclination of the landfill. Managing the landfills in this way has led to the stability of slopes being a major concern (Singh and Murphy, 1990). The stability analysis for MSW landfill is normally performed following the methods described earlier for soil stability using the limit equilibrium analysis (Stark et al., 2000; Seed et al., 1990; Gharabaghi et al., 2008). Previous research results show that the shear strength of MSW is sufficient as the evidence of stable cut slope in MSW landfill. A rotational stability failure is not likely to occur within the waste mass unless the upper surface of the waste is very steeply graded. However, with contributory factor or triggers such as high leachate level, weak foundation soil or failure along weak geosynthetic interfaces, deep-seated catastrophic failures may happen. The evidence of many slope failures occurred at the waste/soil interface, within the waste, in the foundation soil, along the interface between the geomembrane liner and the native soil, and due to operational errors, during the last two decades confirms that it requires the continuous evaluation of slope safety during the construction practices and operating procedures at MSW landfills.

In general, the two-dimensional analysis is appropriate for slope design and yields a conservative estimate for the factor of safety (FoS) because the end effects (resisting forces along the lateral sides of the slide mass) are not included (Duncan, 1996; Stark and Edil, 1998). Although two-dimensional analyses are helpful for designing most slopes, they are not applicable to many situations, such as slopes with transverse loads or concentrated loads, complex geometry in the lateral direction, varying piezometric level due to the underlying
topography, anisotropic and heterogeneous material, etc. A number of three-dimensional slope stability methods and computer programs have therefore been developed; however, a widely accepted three-dimensional stability analysis method is not yet available for practicing geotechnical engineer due to the lack of validity with field case histories (Stark and Edil, 1998).

The most common approach to three-dimensional slope stability analysis is still the limit equilibrium method (LEM), which is usually a direct extension of various two-dimensional methods (Lam and Fredlund, 1993). All these methods divide the failure mass into a number of columns with vertical interfaces and use the conditions for static equilibrium to find the factor of safety. However, limit equilibrium procedures for three-dimensional slope analyses are generally less well-developed and less used than that for two-dimensional problems. The probable reasons are summarized by Loehr et al. (2004). Wei et al. (2009) pointed out that the lack of a suitable method for locating the critical general three-dimensional failure surface is one of the major limitations of the three-dimensional LEM. The published applications of three-dimensional analyses are mainly focused on the back-analysis of mobilized shear strength of the materials involved in a slope failure where the sliding mass geometry is already known (Seed et al., 1990; Stark and Edil, 1998; Stark et al., 2000). Optimization techniques have been successfully used in locating critical slip line in two-dimensional slopes (Pham and Fredlund, 2003; Zheng et al., 2006; Cheng, 2003.), while it is not an easy task to apply them in three-dimensional cases.

In recent years, there have been great developments for slope stability analysis using finite element tools. The most popular one is the strength reduction method (SRM). The advantages and procedures of SRM are fully illustrated in Griffiths and Lane (1999) for various two-dimensional slope analyses. Krahn (2003) mentioned that the major drawback of SRM is that it does not give a direct indication of the margin of safety, and nonconvergence is a somewhat uncertain criterion for determining the point of failure and defining the margin of safety. Extension of the SRM to three-dimensional analysis appears to be simple in principle, but current commercial programs which can provide the three-dimensional SRM option are much fewer than the two-dimensional SRM analysis, and many practical limitations have been found when using them (Wei et al., 2009). Wei et al. (2009) gave an example of the difficulty in choosing the proper convergence criterion and the maximum number of iterations in the three-dimensional SRM analysis by showing that FoS for the vertically cur slope varies from 0.9375 to 2.3125 with various convergence criterion and maximum number of iterations.

Hybrid methods, combining the benefits of a finite element stress-strain analysis with the families of traditional method of limit equilibrium analyses, were highly recommended by Lam and Fredlund (1994) and Krahn (2003). Such a combination provides greater certainty and flexibility regarding the internal distribution of stresses within the soil mass because assumptions associated with LEM related to the interslice or intercolumn forces are no longer
required. With this procedure, the stress state at any point within the sliding mass is obtained directly from the finite element analysis. Fredlund et al. (1999) demonstrated the detailed procedures for such a technique in a two-dimensional slope stability analysis with circular slip surface. Krishnamoorthy (2007) proposed a method of combining the finite element stress analysis with Monte Carlo method to locate slide surface for two-dimensional slopes. Still, the most difficult task of locating the slip surface in three-dimensional space is not diminished. Gitirana et al. (2008) mentioned that optimization techniques can be applied in hybrid method for determining the three-dimensional critical slip surface, however, the authors only illustrated examples of FoS calculation for slip surfaces with known shapes and positions.

A type of quasi-three-dimensional analysis method is also available to compute FoS for three-dimensional slope in the practice using two-dimensional analysis results. Sherard et al. (1963), Lambe and Whitman (1969), Baligh and Azzouz (1975) proposed various combining (weighting) method for computing an 'equivalent' three-dimensional FoS from the individual result of conventional two-dimensional analysis for a series of evenly spaced cross sections within the potential sliding body. Loehr et al., (2004) extended the existing quasi-three-dimensional slope stability analysis method by weighting FoS from two-dimensional analysis based on the total equilibrium shear force along each two-dimensional slip surface and correcting the real area of slip surface from the projected area. The accuracy of this method is proved through comparing with rigorous three-dimensional analyses for the predetermined sliding geometry problems.

For geotechnical engineers, there is still a demanding need for simple methods, easily undertaken and rational enough for routine use, to calculate FoS for three-dimensional slopes. The objective of this paper is to evaluate the stability of an active MSW landfill in Spain using a hybrid quasi-three-dimensional method. As stated in Loehr et al., (2004), the task of searching for the critical slip surface in three dimensions is a difficult problem for all three-dimensional stability methods. Therefore, the hybrid quasi-three-dimensional method presented in this paper tries to give a reasonable estimate of FoS without involving the most tedious work of searching the critical slip surface in three dimensions. It locates the possible three-dimensional slide mass by trying all the possible combinations of the individual critical slip surfaces among a series of two-dimensional cross sections. Although the method employs several assumptions and is, thus, approximate and semi-empirical, the procedure is found useful in preliminary estimate of the approximate position and extent of the critical three-dimensional slide mass. It serves as a simple method for estimating the magnitude of three-dimensional FoS when a more rigorous procedure is not available. Engineers can then devote his or her attention to the most important and most difficult issues involved in the analyses of slope stability - those of defining geometry, shear strengths, unit weights, and water pressures, and of determining the possible uncertainties in these quantities (Duncan, 1996).
The most challenging work in the stability analysis for MSW landfills is to appropriately determine the shear strength parameters and unit weight for the designed MSW landfill. Summary of these parameters can be found elsewhere (Starkand and Huvaj-Sarihan, 2009; Singh and Murphy, 1990; Kavazanjian et al., 1995; Eid et al., 2000; Fassett et al., 1994; Manassero et al., 1996; Van Impe, 1998; Dixon et al., 2005; Powrie and Beaven, 1999; Zekkos, 2005). In the finite element stress analysis, a viscoelastic model was used to reflect the large and long-term settlement happened with the process of biodegradation. In this way, dynamic monitoring of slope stability in the long term becomes possible by continuously adjusting the shapes of the slope. Furthermore, influences of time elapsed, leachate level as well as unit weight of waste on FoS were also investigated to supply guidelines for construction practices and operating procedures in the case landfill.

5.2 Slope Stability Analysis

Several most difficult tasks for evaluating slope stability in MSW landfills include accurate determination of shear strength parameters and unit weight of waste. Although significant scatter exists in these parameters due to the natural characteristics of MSW, considerable efforts have been made toward finding generic rules for these parameters of MSW. These findings are especially useful when there lacks information of the site. Moreover, the lower and upper bounds of these published values can be used to obtain the range of safety factors for slope stability analysis in MSW landfills.

5.2.1 Shear strength parameters

Mohr-Coulomb failure criterion has been extensively applied to describe shear strength of MSW, though its validity has not yet been proved (Singh and Murphy, 1990; Fassett et al., 1994). The effective shear strength parameters are expressed as a function of cohesion, \(c'\) and friction angle, \(\phi'\). Determination of these two parameters for MSW is extremely difficult. The published shear strength data of MSW vary widely, with friction angles between 0°-53° and cohesion varying from 0 to 100 kPa. The significant scatter in reported values is due to the non-standardized sampling methods, various sample size, different shear displacement or axial strain imposed during the laboratory shear testing, as well as many other complex factors related to MSW properties, such as waste type, composition, compaction, daily cover, moisture conditions, age, decomposition, overburden pressure, etc (Stark and Huvaj-Sarihan, 2009; Eid et al., 2000). Testing methods used to obtain shear strength of MSW, including field and laboratory testing, and back-analysis of failed waste slopes. The published data were summarized by Eid et al., (2000), Dixon et al. (2005) and Gharabaghi et al. (2008).
The stress-dependent nature of the Mohr-Coulomb strength envelope of MSW was discussed in detail recently by Stark and Huvaj-Sarihan (2009). Furthermore, a recommended bi-linear strength envelope for deep landfills (greater than 15 m in depth) was proposed considering the non-linear increase in shear strength with increasing normal stress. For normal stresses less than 200 kPa, \( c' = 6 \text{kPa} \) and \( \phi' = 35^\circ \) are recommended which are consistent with what suggested by Eid et al. (2000) but with a smaller \( c' \). For normal stresses greater than or equal to 200 kPa, the recommended strength envelope changes to \( c' = 30 \text{kPa} \) and \( \phi' = 30^\circ \).

Most of the currently available shear strength envelopes are plotted in Figure 5-1. Except for the envelope of Eid et al. (2000) which represents the upper bound and the envelope of Manassero et al. (1996) which is the lower bound, the other envelopes exhibit nearly the same trend, especially under higher normal stresses.

![Figure 5-1: Strength envelopes (after Stark et al., 2009).](image)

### 5.2.2 Unit weight

In-place unit weight of MSW determines the stress field within the waste mass. It is an important parameter in performances assessment of MSW landfills, but significant uncertainty exists regarding its value (Fassett et al., 1994; Dixon et al., 2005). Unit weight values vary significantly among different sites and even within the same site. Published values scatter in a quite wide range, from 300 kg/m\(^3\) to more than 2000 kg/m\(^3\). The unit weight is affected by
compaction effort and layer thickness, the depth of burial (i.e. overburden stress) and the amount of liquid present (moisture content), etc. Unlike soils, the unit weight of MSW also varies significantly because of large variations in the waste constituents (e.g. size and density), state of decomposition and degree of control during placement (such as thickness of daily cover or its absence) (Dixon et al., 2005). Kavazanjian (2006) pointed out that most engineering analyses underestimate the in-place unit weight of MSW as a result of lack of field measurements and the prevalence of estimates based upon operator values. It is extensively recognized that methods of in situ large-scale measurement are more reliable, including test pits near the surface and large-diameter boreholes (Zekkos et al. 2006).

The trend of increasing unit weight with stress level has been observed by several researchers (Powrie and Beaven, 1999; Gourc et al., 2001). Power equations for estimating profiles of dry density and saturated density of MSW with depth were supposed by Powrie and Beaven (1999) as illustrated in Figure 5-2. The current study of Zekkos et al. (2006) concluded that individual landfill does have a characteristic unit weight profile versus depth in which the MSW unit weight increases with depth from a characteristic initial value. A hyperbolic equation was proposed consequently to describe the relationship between MSW unit weight and depth. Figure 5-2 also presents the characteristic unit weight profiles for three categories developed by Zekkos et al. (2006), namely MSW landfill with 'low', 'typical' and 'high' compaction effort and soil cover.

5.2.3 Hybrid method for two-dimensional slope stability analysis

In the hybrid method for two-dimensional slope stability analysis, the factor of safety is commonly defined as

\[ FoS = \min_{l \in L} \frac{T_f}{T} \]  

\[ T = \int_\ell \tau d\ell, \quad T_f = \int_\ell \tau_f d\ell \]  

where \( L \) is a set of potential slip lines and \( l \) represents a certain slip line in set \( L \), \( L \) can be composed of straight lines or circular arcs specified by the analyst; \( \int_\ell \) means the stresses are integrated over the length of the slip line \( l \). \( T \) and \( T_f \) are total mobilized shear force and shear strength along the slip line \( l \), respectively; \( \tau \) and \( \tau_f \) denote the mobilized shear stress and shear strength along the slip line \( l \), respectively.
The basic information obtained from a finite element analysis is stress state \((\sigma_x, \sigma_z, \tau_{xz})\) at Gaussian points for each element. After the stress results are exported into a conventional slope stability analysis code, the normal stress and mobilized shear stress can be computed at any point along an arbitrary slip line. The detailed procedure for such an analysis can be found in Krahn (2003) and Fredlund et al. (1999).

### 5.2.4 Hybrid method for quasi-three-dimensional slope stability analysis

There is one type of quasi-three-dimensional slope stability analysis method in which the three-dimensional factor of safety is obtained using two-dimensional procedures of analyses of a series of evenly spaced cross sections through the slide mass, aligned with the direction of sliding (Loehr et al., 2004). The whole landfill is divided into a series of slope bands along the \(y\) direction (Figure 5-3). Each band has two vertical parallel lateral surfaces. The two-dimensional analyses are firstly carried out on all the middle cross sections of the slope bands.
A series of evenly spaced cross sections

One slip band with cylindrical geometry

Figure 5-3: Hybrid quasi-three-dimensional slope stability analysis.

If the width of the slope band is sufficiently small, the stresses can be assumed constant in the y direction within the whole slope band and represented by the values computed on the middle cross section. Here, the slip band is defined as a sliding geometry confined by the two vertical parallel lateral surfaces and a base surface. The base surface of the slip band is a surface extended by the slip line \( l \) in the middle cross section along y direction. Figure 5-3 shows an example of one slip band with cylindrical geometry. For an arbitrary slip band, the mobilized shear stress and shear strength along the base surface are computed as

\[
T' = l_k \cdot \int \tau dl, \quad T' = l_k \cdot \int \tau_f dl
\]

(5-3)

where \( l_k \) denotes the width of the slip band; \( T' \) and \( T'_f \) are the total mobilized shear force and shear strength along the base surface.

The mobilized shear stress and shear strength along the lateral surface of an arbitrary slip band are computed as

\[
T'' = \int S ds, \quad T''_f = \int \tau_f ds
\]

(5-4)
where \( \int_s \) means the stresses are integrated over the whole lateral surface of the slip band; \( \tau \) and \( \tau_f \) refer to the mobilized shear stress and shear strength at the lateral surface along the tangent direction of sliding direction; \( T^\prime \) and \( T_f^\prime \) are the total mobilized shear force and shear strength over the lateral surface.

The three-dimension FoS is computed by analysing the mobilized shear stress and shear strength on slide mass which is defined by a spatial geometry consisting of several adjacent slip bands. Three-dimensional factor of safety is expressed as the ratio of the sum of the shear strength to the sum of the mobilized shear forces on two outermost lateral surfaces and all the constituent base surfaces for the critical slide mass:

\[
FoS = \min_{s \in S} \frac{\sum_{i=1}^{m} (T_f^i) + \sum_{j=1}^{2} (T_f^\prime)_j}{\sum_{i=1}^{m} (\tau^i) + \sum_{j=1}^{2} (\tau^\prime)_j}
\]

where \( S \) is a set of potential slide mass and \( s \) represents a certain slide mass in set \( S \); \( m \) represents the total number of the slip bands involved in the slide mass \( s \). \( (T_f^i) \) and \( (T_f^\prime)_i \) are the total mobilized shear forces and shear strength along the base surface of the \( i \)th slip band, respectively; \( (\tau^\prime)_j \) and \( (T_f^\prime)_j \) are the sum of mobilized shear stress and shear strength over the \( jj \)th outermost lateral surface, respectively.

### 5.2.5 Three-dimensional factor of safety for circular slide surface

In the quasi-three-dimensional slope stability analysis, one intensive task is defining the two-dimensional cross sections from the three-dimensional model (Loehr et al., 2004). The task involves building up two-dimensional profiles for each cross section and locating the intersections of each cross section with three-dimensional elements. Each two-dimensional cross section is composed of an assemblage of planar patches. Each patch belongs to a three-dimensional finite element. If the patch is sufficiently small, it can be assumed that stresses are constant within the patch and are signified by the stresses at the centre point of the patch which can be interpolated from stresses at Gaussian points of its parent three-dimensional element.

Duncan (1996) pointed out that unless there are geological controls that constrain the slip surface to a noncircular shape, it can be assumed with little inaccuracy that the critical slip surface is circular. The circular slip surface can be approximated by an assemblage of
linear segments in two-dimensional case. FoS of any cross section is computed as the ratio of the sum of the incremental resisting moment, $M_{res}$, to the sum of the driving moment, $M_{slip}$, along the critical slip line:

$$ FoS = \min_{l \in L} \frac{\sum_{i=1}^{n} (M_{res})_{i}}{\sum_{i=1}^{n} (M_{slip})_{i}} = \min_{l \in L} \frac{\sum_{i=1}^{n} [(c + (\sigma - u) \tan \phi) \cdot l_{i}] \cdot R_{k}}{\sum_{i=1}^{n} (\tau_{i} \cdot l_{i}) \cdot R_{k}} \quad (5-6) $$

where $n$ is the total number of discrete linear segments used to approximate a certain slip line $l$; $\sum$ denotes the sum of all linear segments along the slip line $l$; $l_{i}$ is the length of the $i$th linear segment, $c$ and $\phi$ are shear strength parameters of MSW; $\sigma_{ni}$, $\tau_{i}$ and $u_{i}$ are the normal stress, shear stress and water pressure acting on the $i$th linear segment, respectively; $R_{k}$ is the radius of the slip line $l$. The normal and shear stresses acting on the $i$th linear segment are computed as follows:

$$ \sigma_{ni} = \sigma_{x} \sin^{2} \theta_{i} + \sigma_{z} \cos^{2} \theta_{i} + 2 \tau_{xzi} \cos \theta_{i} \sin \theta_{i} $$

$$ \tau_{i} = \sigma_{x} \cos \theta_{i} \sin \theta_{i} - \sigma_{z} \cos \theta_{i} \sin \theta_{i} + \tau_{xzi} \left( \cos^{2} \theta_{i} - \sin^{2} \theta_{i} \right) \quad (5-7) $$

where $\theta_{i}$ is the inclined angle of the $i$th segment with $x$ axis; $\sigma_{x}$, $\sigma_{z}$ and $\tau_{xzi}$ are the principle stresses at the $i$th linear segment. Each linear segment belongs to one patch of the cross section and consequently belongs to one parent three-dimensional finite element. If the linear segment is sufficiently small, it can be assumed that stresses are constant within the small linear segment which can be signified by stresses at the centre point of the linear segment (Pham and Fredlund, 2003). The stresses at the centre point of linear segment can be interpolated from stresses at Gaussian points of its parent three-dimensional element.

The two-dimensional FoS for each cross section is determined using Equation (5-6). Several critical slip lines for each cross section are located using a trial and error procedure by systematically changing the position of the centre of the circle and the length of the radius. The potential slip surface for each slope band is assumed to be a cylindrical surface extended by the a potential slip line along $y$ direction. The three-dimensional slide surface for any potential slide mass is assumed to be the combination of the base surfaces of all the constituent slip bands. In order to ensure the continuity of the slip surface, shapes and positions of each constituent slip band must be checked. It is necessary to keep several potential slip surfaces for each slope band. If the critical one is found impossible to be connected with the adjacent one, it can be replaced by other candidates.
The total resisting moment at two outermost lateral surfaces for a possible slide mass, denoting as $M_{\text{lateral}}$, is calculated as the sum of the incremental shear strength at the patch along the tangent direction of the lateral surface multiplied by the area of the patch, $S_k$, and further multiplied by the distance from centre of the patch to the rotational centre, $r_k$ (Figure 5-3). Similarly, the total driving moment at lateral surface, denoting as $M_{\text{lateral slip}}$, is computed in terms of shear stress. The total resisting moment and driving moment along the base surfaces of all the constituent slip bands, denoting as $M_{\text{base res}}$ and $M_{\text{base slip}}$, are the product of the force defined in Equation (5-3) and the respective radius of the base surface, $R_k$. The three-dimensional FoS is expressed as the ratio of the sum of the resisting moments to the sum of the driving moments for the critical slide mass:

$$\text{FoS} = \min_{sS} \frac{M_{\text{base res}} + M_{\text{lateral res}}}{M_{\text{base slip}} + M_{\text{lateral slip}}} = \min_{sS} \frac{\sum_{i=1}^{m} \left\{ \sum_{k=1}^{n} \left[ (c + \sigma' \tan \phi) \cdot l_k \right] \cdot R_k \right\} + \sum_{j=1}^{2} \sum_{j=1}^{k} \left[ (c + \sigma' \tan \phi) \cdot S_{kj} \cdot r_{kj} \right]}{\sum_{i=1}^{m} \left\{ \sum_{k=1}^{n} \left[ (\tau \cdot l_k) \cdot R_k \right] + \sum_{j=1}^{2} \sum_{j=1}^{k} \left( \tau_j \cdot S_{kj} \cdot r_{kj} \right) \right\}}$$

(5-8)

where $m$ the total number of the slip bands involved in a certain slide mass $s$; $n$ is the number of linear segment for a certain slip line at the base surface for $i$th slip band; $k$ is the total number of patches on the $jj$th outermost lateral surface; $l_k$ and $R_k$ denote the width and radius of $i$th slip band, respectively; $S_{kj}$ is the area of the $j$th patch; $r_{kj}$ is the distance from centre of the $j$th patch to the rotational centre.

### 5.3 Hybrid quasi-three-dimensional slope stability analyses at a MSW landfill

An example problem using the hybrid quasi-three-dimensional method introduced in the previous section was studied at a MSW landfill in this section.

#### 5.3.1 Introduction of the Coll Cardús landfill

Coll Cardús sanitary landfill is an active municipal and industrial solid waste disposal centre in the region of northern-eastern Spain with an approximate area of 200,000 m$^2$. The waste was directly dumped in the natural valley since the middle of 1980s without using geosynthetic membrane at the base. The waste was deposited from a small bottom layer at the
base up to an extended top layer. The depth of the waste varies with the topography of the valley from 40 m to 70 m. The difference in elevation between the top of the landfill and the toe reaches about 150 m by the end of 2007. The inclination of the front slope near the toe of the landfill is 3.7H : 1V. Leachate is collected and treated in the leachate reservoir located in front of the slope. Figure 5- 4 gives an aerial view of the landfill, together with a section view in the middle of the landfill (Y=4605070 m). Leachate level is controlled within a certain level and its historical highest position is shown in Figure 5- 4. According to the 22-year-construction history, the whole landfill can be divided into six layers. The detailed layer information is listed in Figure 5- 5.

![Figure 5- 4: Aerial and section view of coll cardús landfill.](image)
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At Coll Cardús MSW landfill, the valley shape of the waste mass forms a confining effect on the refuse which may have a significant influence on the stability of the slope. Furthermore, the layered construction procedure and time varying stress-strain behaviour of MSW require the incremental, non-linear finite element analysis to accurately compute the state of stress within the waste mass. In order to compute FoS of slope and locate roughly the position of the potential slide mass, slope stability analysis using hybrid quasi-three-dimensional method has been carried out in this study.

<table>
<thead>
<tr>
<th>Number</th>
<th>Duration of construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt; layer</td>
<td>5 years</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt; layer</td>
<td>3 years</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt; layer</td>
<td>2 years</td>
</tr>
<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt; layer</td>
<td>2 years</td>
</tr>
<tr>
<td>5&lt;sup&gt;th&lt;/sup&gt; layer</td>
<td>5 years</td>
</tr>
<tr>
<td>6&lt;sup&gt;th&lt;/sup&gt; layer</td>
<td>5 years</td>
</tr>
</tbody>
</table>

Section at Y=4605070

Figure 5-5: Three-dimensional model of the MSW landfill with layer information.

5.3.2 Unit weight of waste from large-scale tests at the case landfill

In order to get an accurate estimation of the unit weight of waste at Coll Cardús landfill, large-scale unit weight test were carried out on the site (Yu et al., 2011). The measured density varies from 1500 kg/m<sup>3</sup> to 2100 kg/m<sup>3</sup> with an average value of 1850 kg/m<sup>3</sup> (blue triangles in Figure 5-2). These unit weight values are obtained without correction of overbore and locate in the higher region of the database collected by Zekko et al. (2006).
Published data on the unit weight of MSW indicate a non-linear relationship between the unit weight and the effective confining stress (Powrie and Beaven, 1999; Kavazanjian et al., 1999; Zekkos et al., 2006). The stress-dependent density profiles adopted in this paper are parallel to the curves suggested by Powrie and Beaven (1999), but consider an increase in the density value of 1000 kg/m$^3$ (Figure 5-2). Dry density and saturated density are expressed respectively as

\[
\gamma_{\text{dry}} = 155.4 \times 1000^{0.248} \times (\sigma_v')^{0.248} + a \quad (\text{kg/m}^3)
\] (5-9a)

\[
\gamma_{\text{sat}} = 669.1 \times 1000^{0.0899} \times (\sigma_v')^{0.0899} + a \quad (\text{kg/m}^3)
\] (5-9b)

where $\sigma_v'$ is the effective vertical stress (MPa) and $a$ is taken to be 1000 kg/m$^3$. Density data from eight in situ unit weight tests locate within the range surrounded by these two lines as shown in Figure 5-2. Bulk density of the waste within a finite element is calculated based on the saturation degree $S_l$ as

\[
\gamma_{\text{bulk}} = S_l \times \gamma_{\text{sat}} + (1 - S_l) \gamma_{\text{dry}}
\] (5-10)

5.3.3 Mechanical and shear strength parameters

MSW exhibits obvious time-dependent behaviour. The constitutive model used in the finite element analysis is a non-linear K-H rheological model (VOIGT form of the standard linear solid model as illustrated in Figure 3-2), which can be used to describe the long-term deformation of the waste (Yu et al., 2009 & 2010). The compressibility of solid matrix is controlled by effective stress $\sigma' = \sigma - u$, where $\sigma$ is the total stress, $u$ is the excess pore pressure. When there is no external load applied on the landfill, the total stress will be gravitational stress caused by the self-weight of waste. In K-H rheological model, the first Hooken spring with a stiffness of $E_0$ is used to simulate the magnitude of primary deformation during construction of the subsequent layer. Another Kelvin element consisting of two parallel basic elements, namely the second Hooken spring, characterized by its modulus $E$, and a dashpot, characterized by its viscosity $\eta$ is used to simulate the secondary deformation due to the biodegradation process. The parameter $\eta$ controls the decreasing rate of system stiffness which is related to the degradation rate of refuse. Waste deposited earlier has different waste age and decay rate from the waste deposited later. Therefore, each layer has its own starting time of biodegradation, $t_0$, as a result of different waste age in each layer.

The bi-linear strength envelope recommended by Stark et al. (2009) is used for shear strength parameters in this study (Figure 5-1).
5.3.4 Numerical tools used in the slope stability analysis

The three-dimensional model of Coll Cardús landfill is built up using the pre-processor tool, GID8.0. The whole domain is discretized with a total of 15,813 linear tetrahedral elements (Figure 5-5). The finite element analysis is carried out using CODE_BRIGHT Beta3, which was developed for calculating displacements, liquid pressure, gas pressure, temperature and salt content for boundary value problems in saturated or unsaturated soil (Olivella et al., 1996). Stress-strain relationship for the K-H rheological model is implemented into the source code. After running the finite element analysis, stress results at all the nodes are automatically exported to an external module, named 'HERA', for slope stability analysis. The slope stability module contains geometric modelling scheme designed for establishing two-dimensional profiles and discretization for all cross sections from the parent three-dimensional model, and FoS calculation based on the hybrid quasi-three-dimensional method.

The layered construction process and variations of leachate level during the emplacement of waste are taken into account in the finite element analysis. All the elements are divided into six groups representing six layers, and are activated sequentially in the calculation. After the construction of each layer, stresses are kept and displacements are restored to zero. Leachate table increases to a new height before the construction of the next layer.

At Coll Cardús landfill, there is no weak soil foundation and geosynthetic membrane at the base of the landfill, therefore, only circular slip surfaces within the waste body are considered. All the parameters used in the analysis are listed in Table 5-1. Yu et. al. (2010) has already obtained values of the Young’s modulus and residual modulus for Coll Cardús landfill after back analysing field settlement measurements which will be used in the stress analysis.
Table 5-1: Parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus during construction period, $E_{0_cons}$</td>
<td>0.7 MPa</td>
</tr>
<tr>
<td>Poisson ratio during construction period, $\nu_{cons}$</td>
<td>0.45</td>
</tr>
<tr>
<td>Young’s modulus after deposition, $E_0$</td>
<td>7 MPa</td>
</tr>
<tr>
<td>Poisson ratio after deposition, $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Residual modulus of waste, $E$</td>
<td>0.5 MPa</td>
</tr>
<tr>
<td>Viscosity of waste, $\eta$</td>
<td>$3.0864\times10^8$ MPa·s</td>
</tr>
<tr>
<td>Starting time of biodegradation, $t_0$</td>
<td></td>
</tr>
<tr>
<td>1$^{st}$ layer</td>
<td>5 years</td>
</tr>
<tr>
<td>2$^{nd}$ layer</td>
<td>8 years</td>
</tr>
<tr>
<td>3$^{rd}$ layer</td>
<td>10 years</td>
</tr>
<tr>
<td>4$^{th}$ layer</td>
<td>12 years</td>
</tr>
<tr>
<td>5$^{th}$ layer</td>
<td>17 years</td>
</tr>
<tr>
<td>6$^{th}$ layer</td>
<td>22 years</td>
</tr>
<tr>
<td>Initial porosity</td>
<td>0.45</td>
</tr>
</tbody>
</table>

$^a$based on gas generation rate constant $\lambda=1.62\times10^{-9}$ s$^{-1}$ (0.05 year$^{-1}$, default value suggest by Oweis (2006), $\eta=E/\lambda=0.5/1.62\times10^{-9}$ s$^{-1}=3.0864\times10^8$ MPa·s (Yu et al., 2010)

5.4 Results of slope stability analyses

In three-dimensional finite element analysis, the most critical case, after construction of the 6$^{th}$ layer (at closure) with leachate table at its highest level, is considered. The displacement vector maps in three-dimensional space and at the middle cross section ($y=4605070$) are shown in Figure 5-6. The graph shows that large displacement happens in the area containing the front slope during the construction. The displacement vectors indicate the potential sliding direction of the landfill, that is, the waste in the area of the front slope moves from upper periphery towards central and lower right.

After the three-dimensional finite element analysis, all the mesh and stress information is exported automatically to the module of slope stability analysis. The landfill was divided uniformly by 40 cross sections between 4604900 m < $y$ < 4605200 m. The width of each slope band is about 7.7 m. Two-dimensional slope stability analyses are firstly carried out on each cross section. Besides the critical slip circle with the minimum FoS, the other nine worst slip circles are kept as candidates for each cross section. The minimum FoS at forty cross sections are shown in Figure 5-7. The saddle-shaped FoS curve along y direction implies that the potential slide mass exists in the middle area. The positions and shapes of the critical slip circles at all the cross sections are illustrated in Figure 5-8.
Figure 5-6: Results of displacement vector from three-dimensional finite element analysis at the time of closure (t=22 years): (a) Three-dimensional model; (b) Section of y=4605070 m.
It is also observed that the lowest two-dimensional SoF is found at the 20th section of \( y = 4605046 \) m with a value of 1.42. After examining the shape and position of the critical circle in each section, it is found that the critical circle at the 20th section is disconnected with its neighbours. Deep seated slip circles were found at sections between the 15th and 26th sections except at the 20th section, which has a shallow and small critical slip circle (red circle in Figure 5-8). For the continuity of the three-dimensional slide surface, the disconnected slip circle is replaced by another slip circle (marked in red dot in Figure 5-7 and blue circle in Figure 5-8) after searching from the rest nine candidates.

Figure 5-7: Factor of safety at 40 vertical sections.
By analysing the possible combinations among these forty cross sections using the critical circle, the most dangerous slide mass is located in the area of $2500 \text{ m} < x < 2740 \text{ m}$ and $4605000 \text{ m} < y < 4605108 \text{ m}$ with a safety factor of 1.81 (shaded area in Figure 5- 7). The dimension of the slide mass is approximately 230 m in length, 108 m in width and 100 m in height. The three-dimensional view of the most dangerous slide mass is presented in Figure 5- 9. It can be seen that there is some discontinuity at both ends. But at least, the slide mass is quite continuous for the deep seated central slide body (between section 15 to section 26). In reality, the central slide body may be capped by conical ends. All the other potential slide masses include the same central slide body and extended shallow slide surfaces at both ends. The extended part does not change FoS very much, which varies from 1.81 to 1.84. Although the sliding surface at both ends cannot be accurately positioned, the central rotational surface of the deep seated slide mass can be roughly located.

In order to investigate the three-dimensional effect on the FoS, a separate two-dimensional model is established for the central section of $y = 4605070 \text{ m}$. The two-dimensional domain is discretized with a total of 3165 linear triangle elements. In the base case, the same critical situation as in three-dimensional analysis is considered: at the time of closure and with leachate table at its highest level. Figure 5- 10 gives out ten most dangerous slip surfaces. All these potential slip surfaces are located in the front slope, about 40 m deep within the landfill. The computed FoS for the most dangerous slip circle is 1.213 (red circular

Figure 5- 8: Critical slip circles at 40 vertical sections.
line in Figure 5-10). This value is one third lower than three-dimensional FoS of 1.81 because the confining effect of the valley-shaped geometry of the landfill as well as the resistances at two lateral surfaces of the slide mass are taken into consideration in the latter.

Figure 5-9: Three-dimensional view of the potential slide mass.
The settlement of MSW landfill can continue over an extended period with a final settlement that can be as large as 30-40% of the initial fill height (Ling et al., 1998). The large, long-term settlement is due to the biodegradation of waste. With the settlement continuing, the shape of the landfill changes significantly which may influence the FoS of the slope. The management of MSW landfill requires that the slope stability should be controlled not only during construction, but also after closure in the long term. In order to investigate the variation of FoS with time at Coll Cardús landfill, the two-dimensional slope stability analysis in the base case is extended to several cases with different time elapsed after closure, namely 20, 50 and 100 years. In all these cases, the leachate level is conservatively assumed to be...
constant at its highest level. During the finite element analysis, all the element coordination is updated after each time step in order to reflect the continuous deformation of the landfill. Figure 5-10 illustrates the evolution of the surface of the landfill at the time of 20, 50 and 100 years after closure, together with the evolution of the most critical slip circle. The largest settlement at 100 years after closure reaches 26 m which is equivalent to 40% of the waste depth at that spot. The front slope flattens not much because the waste at that area was deposited in the early 1980s and most of the biodegradation settlement has already finished by the time of closure. FoS varies between 1.2-1.31 and does not vary much with time because in this study, two key parameters, unit weight and shear strength, are determined by effective stress and not considered to vary with time. Furthermore, the constant leachate level counteracts the minor increase of FoS due to the little flattening of the front slope.

The leachate level is relatively high during construction of the landfill. After closure, the constructor plans to lower the leachate level through pumping. The effect of leachate level on FoS is studied through various cases by lowering the leachate level uniformly and gradually between 10 m up to 40 m from its highest level. The results in Figure 5-12 indicate that FoS increases rapidly with lowering the leachate table by the first 20 meters. Afterwards, lowering leachate level cannot increase FoS anymore because a newly-formed shallow slip circle comes up in the upper landfill, which is located above the leachate level and will no longer be affected by the position of the leachate level.

Unit weight of waste is a big uncertainty in the slope stability analysis for MSW landfill. By varying the value of $a$ in Equation (5-3) from zero to 1000 kg/m$^3$, the influence of waste unit weight on the FoS is investigated. The results in Figure 5-13 show that FoS increases monotonously with density of waste. The position of the most dangerous slip surface does not change. The strength parameters are considered in this study increasing linearly with the effective stress, and the effective stress is proportional to the density of waste. Thus, the denser MSW has a higher driving force, it also develops higher resisting forces. Increasing the initial density increases the stability of the MSW landfill at a given slope (Boutwell, 2002).
5.5 Summaries and conclusions

A hybrid method for quasi-three-dimensional slope stability analysis was carried out in this study at a Spanish municipal solid waste landfill. This method computes an 'equivalent' three-dimensional factor of safety from results of two-dimensional analyses for a series of evenly spaced cross sections within the potential sliding body, aligned with the direction of sliding. The two-dimensional analysis at each cross section is realized by combining finite element stress analysis with the concepts of limit equilibrium method. In the finite element
stress analysis, variations of leachate level during layered emplacement of waste were taken into account. Time-dependent deformation of MSW based on viscoelastic model (K-H rheological model) was used to reflect the effect of the process of biodegradation.

Without weak foundation soils and geosynthetic membranes at the base, only circular slip surface was considered at the case landfill. Two-dimensional slope stability analysis at the section containing the steepest part of the front slope with the highest leachate level at the time of closure results a FoS of 1.2. The potential slip surface is located in the front slope, about 40 m deep within the landfill. The potential three-dimensional slide mass, with a dimension of approximately 230 m in length, 108 m in width and 100 m in height, is located through three-dimensional analysis with a FoS of 1.8. FoS from three-dimensional analysis is 50% higher than that from two-dimensional analysis because the confining effect of the valley-shaped geometry of the landfill as well as the resistances at two lateral surfaces of the slide mass were taken into consideration. Results show that the hybrid method for quasi-three-dimensional slope stability analysis based on FEM results is easily manipulated and can serve as an engineering tool in the preliminary estimate of the FoS as well as the approximate position and extent of the potential slide mass.

Influences of time elapsed after landfill closure, leachate level as well as unit weight of waste on FoS were investigated based on two-dimensional slope stability analysis at the central section. The surface of the landfill was dynamically monitored by continuous updating the element coordination in the finite element analysis. Results show that factor of safety does not vary much with time, varying between 1.2-1.31. One reason is because two key parameters, unit weight and shear strength, are determined by effective stress and not considered to vary with time in this case study. The other is the shape of the front slope does not flatten much after closure because the waste at that area was deposited in the early 1980s and most of the biodegradation settlement has already finished by the time of closure. FoS is found to increase rapidly with lowering the leachate level by the first 20 meters. Afterwards, lowering leachate level cannot increase the FoS anymore because a newly-formed shallow slip surface comes up in the upper landfill, which is located above the leachate level and will no longer be affected by the position of the leachate level. In this study, strength parameters are assumed increasing linearly with the effective vertical stress which is determined directly by the unit weight of waste. Therefore, increasing the initial density of waste increases monotonously the slope stability.
CHAPTER 6

Numerical analyses of a leachate pumping test at a MSW landfill

Abstract

This chapter presents the results of a long-duration pumping test with variable pumping rate performed in a MSW landfill. Numerical simulations were conducted in order to get a better understanding of the coupled hydro-mechanical behaviour of waste. Measurements of the surface settlement near the pumping well reveal that six times more settlement was triggered when the pumping rate increased from 0.2 L/s to 0.52 L/s after 12 days since pumping started. Delayed rebound of landfill surface was observed six days after the pumping stopped. These phenomena indicate the possible creep and biodegradation-induced deformation of waste. Numerical simulation using viscoelastic model was conducted in order to get a better understanding of the coupled hydro-mechanical behaviour of waste during pumping. Large numbers of parameters are involved in the numerical simulation due to the consideration of water retention capacity of waste above the leachate level. After ruling out some insensitive parameters from preliminary sensitivity study, several key parameters were adjusted by trial and error to find an agreeable match between measured draw-down curves and simulated results at both the pumping well and two observation wells. The comparison indicates that the conventional viscoelastic model used in the numerical simulation is capable of reproducing the drawdown-time curves and predicting the ultimate magnitude of surface settlement, but not adequate to reproduce the time evolution of the settlement.

Keywords: municipal solid waste; pumping test; hydraulic conductivity; viscosity, coupled hydro-mechanical analysis.
6.1 Introduction

Modern municipal solid waste (MSW) landfills are required to install in situ leachate collection and removal (LCRs) systems to control leachate head mounded over the bottom liner and therefore assure the safety of the landfill. Pumping of leachate from wells is a convenient method to limit the hydraulic head within a certain level, especially in the event of LCR system failure. In addition, the fact that leachate removal increases effective stress isotropically within MSW landfill benefits to many landfill engineering aspects, including slope stability, landfill capacity and geomembrane integrity. Furthermore, modern technique such as moisture addition for bioreactor landfilling requires comprehensive knowledge of hydraulic characters of waste. In bioreactor landfills, the collected leachate is pumped back into the waste causing accelerated waste decomposition and gas production. In the design of whether leachate extraction systems or leachate circulation systems, proper assessment of hydraulic conductivity of waste is a key element because it governs the movement of leachate in the landfills. However the determination of its value is challenging owing to the widely varying permeability values caused by the high heterogeneity nature of waste, placement controlled structure of MSW, time-dependent properties, etc.

Methods used to evaluate hydraulic conductivity include laboratory and in situ methods. Detailed summarization of the range of hydraulic conductivity values reported by many of the previous studies utilizing laboratory or field methods can be found in Shank (1993), Jain et al. (2006) and Reddy et al. (2009). Undisturbed samples in MSW landfills are nearly impossible to get. Remolded samples at the small scale prepared in conventional laboratories are considered not representative. Test apparatus of large sizes especially designed for waste testing are expensive to construct and operate, hence very few researchers tend to test on extensively large-scale waste samples. A constant head flow test was carried out on a waste sample of 2 m in diameter and up to 2.5 m height by Powrie and Beaven (1999). The hydraulic conductivity for the fresh crude domestic waste was reported to vary between $3.1 \times 10^{-6}$ and $2.8 \times 10^{-5}$ m/s, under an average stress of 120 kPa (Powrie and Beaven, 1999). The stress dependency of waste hydraulic conductivity is observed by Reddy et al. (2009) with a trend of decreasing hydraulic conductivity with increasing dry unit weight using a large-scale rigid-wall permeameter apparatus (waste sample with 30 cm in diameter and 30 cm in height).

Even for the extensively large-scale laboratory apparatus mentioned above, it is still difficult to integrate the complex environment encountered in the MSW landfills (e.g., discontinuities, irregularities, intermediate soil layers, perched water, etc.). A few investigators have turned to field methods (surface infiltration pit/pond, open-end borehole, pumping test, borehole permeameter test, etc.) to develop more representative estimates of in situ hydraulic conductivity. However, there is limited information in the literature on MSW
hydraulic conductivity measured \emph{in situ} and therefore the current understanding is incomplete (Dixon and Jones, 2005). Oweis et al. (1990) estimated hydraulic conductivity in the range of $9.4 \times 10^{-6}$-$2.46 \times 10^{-5}$ m/s by conducting a leachate pumping test for a MSW landfill with an average saturated thickness of 9.15 m. Wysocki et al. (2003) reported the hydraulic conductivity from $1.2 \times 10^{-7}$ to $6.3 \times 10^{-6}$ m/s based on a variety of pump tests. Jain et al. (2006) found the saturated hydraulic conductivity ranging from $5.4 \times 10^{-8}$ to $6.1 \times 10^{-7}$ m/s after conducting borehole permeameter tests at 23 locations. The authors resulted the relatively lower values of hydraulic conductivity to the new, well-compacted waste and impediment to liquid flow created by the presence of entrapped gas phase. Bleiker et al. (1993) had an opportunity to measure directly the in-situ vertical refuse hydraulic conductivity at the Brock Waste landfill. A value as low as $10^{-8.2}$ m/s was estimated by measuring the increase of chloride concentration over time in the groundwater subdrain installed under the site.

A constant rate pumping test with steady-state conditions in MSW landfills normally is not easy to achieve. Paper, plastic, fibers and other debris tend to plug the screen and gravel pack of the pumping well. Experienced practitioners have limited confidence for evaluating hydraulic properties from the existing theoretical methods for unsteady-state conditions, e.g. Theis (1935) and Cooper and Jacob (1946). The S shape which is believed to appear in plots of logarithm of drawdown versus logarithm of time for unconfined aquifers is not frequently obtained from the actual pumping test. The obvious underestimation of $S_y$ from curve fitting methods is attributed to ignoring the highly nonlinear unsaturated $K$ function and nonlinear capillary retention curve above the water table (Chapuis et al., 2005).

In addition, a structure of sub-horizontal layers induced by the placement of waste in layers as well as the utility of daily cover soil results in the anisotropic hydraulic properties for MSW landfill with higher permeability in horizontal direction (Dixon and Jones, 2005). Under the circumstances which deviate from the ideal conditions for deriving the theoretical predictions, numerical models will serve as an appropriate method for studying the water flowing behaviour. Chapuis et al. (2005) achieved an excellent fit for both steady- and unsteady-state conditions for a pumping test in an unconfined aquifer after introduction of a two-layer zone and consideration of anisotropy.

A leachate pumping test was conducted at an active MSW landfill in this study with the purpose of investigating the hydraulic properties of waste. Measurements of surface settlement induced by pumping around the pumping well are also available for investigation of the mechanical behaviour of waste. Numerical analyses for the pumping test were performed in order to get a better understanding of the coupled hydro-mechanical (HM) behaviour of waste, which consider the water release due to the mechanical deformation, the water retention capacity of waste and variations of the relative permeability with water saturation. This paper presents field data and numerical method used to obtain hydraulic and
mechanical parameters of waste, which provide a good match between field data and numerical results.

### 6.2 Description of the pumping test

The pumping test described in this paper was performed at Coll Cardús landfill, an active municipal and industrial solid waste disposal centre in the region of Catalonia, eastern Spain. Detailed information can be found in Yu et al. (2010 & 2011). Figure 6-1 shows the plan and section view of the landfill.

Several types of in-situ test were carried out in 2007 at Coll Cardús landfill in order to evaluate the physical properties of the waste. A long-duration leachate pumping test was performed to investigate the hydraulic properties of waste and with the purpose to supply information for an optimal leachate discharging plan after closure. Although plate loading test is considered not suitable for obtaining shear strength parameters in MSW landfills because the failure plane cannot be located (Eid et al., 2000), it does supply information regarding the mechanical behaviour of waste. Two surface plate loading tests were conducted as part of this study with the diameter of the plate equal to 600 mm. Waste composition and unit weight are characterized from two large-diameter borehole unit weight tests. Test procedure and results of in situ unit weight test have been described in detail in Yu et al. (2011).

Positions of one pumping test, two plate loading tests and two unit weight tests are marked in Figure 6-1. The location of the pumping test was selected to be representative and to avoid the interferences from the surroundings, but also to meet operational requirements because the landfill was still active then. The depth of the waste at the test location was approximately 50 m. The leachate mounded with an average saturated-thickness of about 40 m. The pumping well and four observation wells were drilled using bucket auger with a net diameter of 760 mm. A thick layer of 20 cm gravel pack (gravel with an averaged diameter of 5 cm) was used to minimize the plugging of the screen by waste debris. Four observation wells, OB1-OB4 were installed at the distance of 4 m, 12.65 m, 16.5 m and 20.4 m from the pumping well, respectively. The spatial positions of the pumping well and four observation wells are illustrated in Figure 6-2, together with the initial leachate level in each well. The pump intake was located at a depth of about 45 m in the pumping well. The pumping rate was controlled by a volumetric meter. The drawdown in all five wells was measured both automatically with piezometers and manually.

At the beginning of the pumping, the leachate pumped out from the pumping well were analysed with pH = 6.11, Eh = -179, conductivity K= 50.1 mS/cm, and the temperature = 54.8 °C.
Figure 6-1: Plan view of Coll Cardús landfill and locations of the in situ tests.
According to the pumping test performed earlier at Coll Cardús landfill, constant pumping rate cannot be maintained with a value of 1.5 L/s because of the plugging of the well screen and the consequent loss of efficiency. Therefore, a constant-rate pumping test was first tried with a selected flow rate of 0.2 L/s. The pumped flow rate was checked three times a day and it was found that the measured flow rate decreases a little with time. The measured pumped flow rate is presented in Figure 6-3. After 6 days, the pumping rate was adjusted manually back to 0.2 L/s. The pumping rate was increased to 0.52 L/s at the 12th day in order to trigger more apparent surface settlement around the pumping well. In the following 10 days, the pumping rate was found not stable and decreasing steadily with time. The recovering period started from the 22nd day and the drawdown record continued until 46 days since the pumping started.

Figure 6-2: Spatial positions of the pumping well P and four observations wells (OB1-OB4) with their initial leachate levels.
The drawdown was measured in the pumping well PW and four observation well OB1-OB4. Measured drawdown-time curves during the whole duration of pumping are recorded in Figure 6-4. Total drawdown in PW, OB1 and OB2, which amounted to 4.58 m, 1.53 m and 0.94 m, respectively, at the end of the 12th day is only about one quarter of that at the end of the 22nd day, which amounted to 15.58 m, 6.73 m and 3.6 m, respectively. OB3 is supposed to be connected with OB4 according to the drilling log because there was borehole collapse during the construction of OB3. Therefore, the drawdown-time curves at OB3 and OB4 are nearly the same. The leachate level in all wells is observed to recover higher than the pretest static level at the end of the recovering stage.

Besides measuring the drawdown in each well, time evolution of the surface settlement caused by the pumping was also recorded. Twenty concrete plates were mounted at both sides of the pumping well at an interval of 2 m along the settlement measuring line as illustrated in Figure 6-1 and Figure 6-C-2 in Appendix 6-C. Ten plates are in the direction parallel the observation wells and the other ten are in the reversed direction. The three-dimensional movement of each concrete plate was recorded intermittently using laser apparatus on the 1st, 6th, 12th, 25th, 28th and 33rd day since the pumping started.
Chapter 6  Numerical analyses of a leachate pumping test at a MSW landfill

Figure 6-4: Measured Drawdown-time curves.

Surface settlement at all concrete plates were measured to keep increasing during the whole pumping period. When the pumping stopped, settlement plates continued to settle for several days more. Measurements at the 33rd day show that the settlement stops and waste started to rebound. The maximum settlement of 0.049 m was recorded at a distance of 2 m from the pumping well. The settlement-time curves at six settlement plates, which are at the distance of 2 m, 10 m and 20 m from the pumping well are plotted in Figure 6-5. Figure 6-5 shows that surface settlement at the end of the 0.2 L/s pumping rate (i.e. at the 12th day), is only about 1/6 of the settlement at the end of the 0.52 L/s pumping rate (i.e. at the 22nd day). The obvious delay of settlement in the early stage and delayed rebound in the recovery stage indicates the viscos behaviour of the waste.

Although concrete plates were mounted at both sides of the pumping well symmetrically, settlement measurements are not symmetric. The plates located in the direction apart from the observations wells settled more than those plates at the other side. The surface settlement profile at the 28th day is presented in Figure 6-6.
Figure 6-5: Measured and simulated time-settlement curves at the distance of 2 m, 12 m and 20 m from the pumping well.

Figure 6-6: Comparison of the measured and simulated final surface settlement around the pumping well.
6.3 Mechanical properties of waste

In order to have a preliminary concept of the mechanical behaviour of waste at Coll Cardús landfill, two plate loading tests using plate with diameter of 600 mm were carried out at the site. The locations of the plate loading tests are shown in Figure 6-1. The pressure-settlement curves for two plate loading tests are presented in Figure 6-7.

![Figure 6-7: Results of two plate loading tests and numerical simulation results for back analysing elastic modulus.](image)

The virgin loading curve depends on the deformability and strength of the waste. Within the elastic domain, the incremental load-settlement law is expressed as (Sánchez et al., 1995):

$$\Delta p = \frac{E_0}{1 - \nu^2} \frac{\Delta s}{B}$$  \hspace{1cm} (6-1)

where \( p \) is the pressure applied on the loading plate (kPa), \( E_0 \) and \( \nu \) are Young’s modulus and Poisson ratio of the waste, \( s \) is settlement of the plate (m) and \( B \) is the diameter of the plate (m).

During the stage of initial loading, Young’s modulus of the waste is equal to 13.46 MPa and 17.06 MPa for two tests, respectively, with an average value of 15.26 MPa (\( \nu \)=0.3). During the stage of reloading, elastic modulus is equal to 33 MPa and 37 MPa, respectively, with an average value of 35 MPa. After the complete unloading, more than 50% irrecoverable settlement happened which indicates that the waste behaves in an elastoplastic, strain hardening way. Elastic modulus calculated is higher than the value of 1-2 MPa reported by
Sánchez et al. (1995) from plate loading tests with diameter of 300 mm, 450 mm and 750 mm. This is probably due to the high bulk density of 18.5 kN/m$^3$ measured at Coll Cardús landfill (Yu et al., 2011) which is much higher than the waste density of 8-10 kN/m$^3$ reported by Sánchez et al. (1995).

Elastic modulus of 9.5 MPa was obtained from three dimensional (3D) numerical simulation of the plate loading test. The 3D model is a cubic with length of 6 m and the waste was treated as an elastic material with Poisson ratio of 0.3. The loading pressure was applied gradually by a rigid plate. The settlement at the centre of the plate is compared with field measurement in Figure 6-7. This value serves as a guideline for choosing parameter values in the following mechanical model.

### 6.4 Estimation of the hydraulic properties using theoretical solutions

The hydraulic properties can be preliminarily estimated by curve fitting of drawdown-time data using Cooper-Jacob semilog plot (Cooper and Jacob, 1946). Under unconfined conditions, gravity drainage of interstices reduces the saturated thickness and, therefore, the computed coefficient of transmissivity of the aquifer. Drawdown data need to be adjusted to compensate for the decrease in saturated thickness if the true transmissivity is to be calculated using the Jacob analysis (Oweis, 1990). The procedure is to fit a straight line to the latter portion of the curve. Transmissivity, $T$, is obtained from the slope of the straight line and specific yield, $S_y$, from the intercept. One example of the graph of corrected drawdown, $s$ (m), versus log $t$ is shown in Figure 6-8.
Figure 6-8: One example of semilogarithm curve of the corrected drawdown versus log t for OB1 during time period 12-22 days (r=4 m) (Drawdown is corrected by $s' = s - s'^2/2b$, where b is the saturated thickness of the aquifer and is supposed to be 40 m in this study).

Table 6-1 gives a summary of the $T$ and $S_y$ obtained from the pumping well, OB1 and OB2 at various pumping stage. $T$ was computed to range between 1.84-9.2 m²/day, with an average value of 4.3 m²/day. The corresponding hydraulic conductivity is approximately $1.3 \times 10^{-6}$ m/s with respect to the saturated-thickness of 40 m. This value locates within the range reported previously from pumping test at MSW landfill (see Table 6-2). The specific yield, $S_y$, calculated from the drawdown plots for observation wells OB1 and OB2, varies between 0.015 and 0.1, with an average value of about 0.05, the same value reported by Oweis (1990) for MSW.

Table 6-1: Hydraulic properties obtained using Cooper and Jacob (1946) formula.

<table>
<thead>
<tr>
<th>Well</th>
<th>Distance (m)</th>
<th>Pumping phase</th>
<th>$T$ (m²/day)</th>
<th>$S_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PB</td>
<td>0</td>
<td>0-6 days</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>12-22 days</td>
<td>1.84</td>
<td></td>
</tr>
<tr>
<td>OB1</td>
<td>4</td>
<td>0-6 days</td>
<td>5.7</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12-22 days</td>
<td>2.35</td>
<td>0.05</td>
</tr>
<tr>
<td>OB2</td>
<td>12.65</td>
<td>0-6 days</td>
<td>9.2</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12-22 days</td>
<td>3.8</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>Averaged value</td>
<td></td>
<td>4.3</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Hydraulic conductivity $K$ (saturated thickness $b$=40 m)</td>
<td></td>
<td>$1.3 \times 10^{-6}$</td>
<td>(m/s)</td>
</tr>
</tbody>
</table>
Table 6-2: Hydraulic conductivity of MSW reported from previous pumping tests.

<table>
<thead>
<tr>
<th>Reported by</th>
<th>Hydraulic conductivity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ettala (1987)</td>
<td>$5.9 \times 10^{-5}$ - $2.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Oweis et al. (1990)</td>
<td>$9.4 \times 10^{-6}$ - $2.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Wysocki et al. (2003)</td>
<td>$1.2 \times 10^{-7}$ - $6.3 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

The interpretation of the drawdown-time curves based on the theoretical solutions does not consider the unsaturated properties of the aquifer material however it is key parameters in a transient process. Compared to the current theoretical methods for transient conditions, the numerical code solves the exact conservation equation without introducing the simplifying assumptions on which the theoretical methods are based. In particular, it does not reduce the storativity $S$ and specific yield $S_y$ to constant number (Chapuis et al., 2005). Furthermore, numerical method is especially helpful for analysing pumping test performed in MSW landfill, which is normally under unsteady-state conditions and involved with heterogeneous material.

### 6.5 Numerical modelling of the pumping test

In this section, the pumping test conditions are modelled numerically using a finite element code CODE_BRIGHT. The code considers the full characteristic functions of the porous media, including the water release due to the mechanical deformation, the water retention relationship between saturation degree of water, $S_l$, and pore water pressure, $P_l$; a relationship between the unsaturated relative permeability, $k_r$, and water saturation, etc.

The geometry of the pumping test at Coll Cardús landfill is idealized with an axisymmetric cylindrical geometry, which is 150 m long and 50 m high (Figure 6-9). A grid configuration with 3404 linear quadrilateral elements and 3525 nodes was used. The waste domain was discretized in fine layers with the flexibility of setting stratified heterogeneity later on. The well is composed of gravel with the hydraulic conductivity several orders higher than that of the refuse. In order to investigate the possible skin effect, a thin skin layer was set between the pumping well and waste. Mechanical boundary conditions include no horizontal movement along the centre axis of the pumping well and no vertical movement at the bottom of the domain. Initial leachate level was assigned horizontally 10 meters beneath the landfill surface. Hydraulic boundary conditions include a time variable pumping rate imposed along a 10-m-long screen at the lower part of the pumping well. No water influx is allowed from the peripheral region. The pumping is roughly divided into four stages: 0-6 days, 6-12 days, 12-22 days and 22-46 days. The measured pumped water flow rate at each stage is linearized for the numerical simulation and illustrated in Figure 6-3.
Chapter 6 □ Numerical analyses of a leachate pumping test at a MSW landfill

6.5.1 Mass balance equation

Conservation of mass for the liquid phase is expressed as

\[
\frac{\partial}{\partial t} \left( \rho_l \phi S_l \right) + \nabla \cdot \left( -\frac{k_i k_r}{\mu} \left( \nabla P_l - \rho_l g \right) \rho_l + \rho_l \phi S_l \mathbf{u} \right) = f^w
\]  

(6-2)

where \( \rho_l \) is the liquid density (kg/m\(^3\)), \( \phi \) is the porosity of the waste, \( S_l \) is the degree of liquid saturation, \( k_i \) and \( k_r \) are intrinsic and relative permeability for the liquid phase, respectively, \( \mu \) is viscosity of liquid (MPa\( \cdot \)s), \( P_l \) is liquid pressure (MPa), \( \mathbf{u} \) is solid velocity vector and \( f^w \) is the source/sink term.

The moisture retained in the waste above the leachate level is a function of the capillary pressure. The retention curve of waste is expressed using van Genuchten model (1980):

Figure 6- 9: Mesh and boundary conditions in the numerical simulation.
\[
\frac{S_l - S_{rl}}{S_{ls} - S_{rl}} = \left[ 1 + \left( \frac{P_g - P_0}{P_0} \right)^{1-n} \right]^{-n}
\]

where \(S_{rl}\) and \(S_{ls}\) are the residual and maximum saturation of liquid, respectively, \(P_g\) is gas pressure and is at atmospheric pressure (0.1 MPa), \(P_0\) is a value related to the temperature and surface tension (MPa), \(n\) is the shape function for the retention curve.

Intrinsic permeability is considered varying with porosity based on the Kozeny’s law:

\[
k_i = k_{i0} \frac{\phi^3}{(1-\phi)^2} \frac{(1-\phi_0)^2}{\phi_0^3}
\]

where \(k_{i0}\) is referenced intrinsic permeability measured with the referenced porosity \(\phi_0\).

Corey’s theory is chosen for the relationship between relative permeability, \(k_r\), and saturation degree of liquid phase:

\[
k_r = \hat{S}^s
\]

\[
\hat{S} = (S_l - S_{rl})/(1-S_{rl})
\]

The balance of momentum for the porous medium reduces to the stress equilibrium equation expressed by the Terzaghi’s concept of effective stress under certain simplifications. Providing an adequate mechanical constitutive model, the stress equilibrium equation is transformed into a form in terms of the solid velocities and fluid pressure (Olivella et al. 1994).

### 6.5.2 Mechanical constitutive model

Surface settlement measured during the first 12 days with a smaller pumping rate of 0.2 L/s is only 20% of that happened during 12 to 22 days with a larger pumping rate of 0.52 L/s (Figure 6-5). The observation that the surface settlement is not proportional to the pumping rate implies that waste behaves in a viscous manner during 12-22 days due to the creep or degradation of waste when the pumping rate is large. Furthermore, the waste matrix exhibits obvious delayed rebound, i.e. 6 days delay in the vicinity of the pumping well, when the pumping stopped at the 22\textsuperscript{nd} day (Figure 6-5). Therefore, it is reasonably believed that waste
matrix firstly behaves in an elastic manner in the first 12 days, when the pumping rate is not large enough to induce matrix collapse and biodegradation does not start. When the pumping rate increases significantly since the 12th day, collapse of the waste matrix or/and creep or/and accompanied biodegradation induced deformation make the waste exhibit obvious viscosity afterwards. For the sake of simplicity, linear elastic model was adopted for the first 12 days in the numerical simulation, while viscoelastic model was applied on the waste after 12 days.

It is well acknowledged that MSW exhibits obvious time-dependent behaviour. The viscoelastic model used in the numerical simulation is a non-linear K-H rheological model (Figure 3-2), which can be used to describe the long-term time-dependent deformation of the waste (Yu et al. 2010 & 2011). Using the K-H rheological model, the total strain is decomposed into:

\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^{ve} \]  

where \( \dot{\varepsilon}^e \) is the elastic strain rate described by the spring with elastic modulus \( E_0 \), \( \dot{\varepsilon}^{ve} \) is viscoelastic strain rate controlled by the Kelvin element, \( \dot{\varepsilon} \) is the total strain rate which is related with solid velocities through the compatibility conditions.

The elastic model is written as:

\[ \dot{\varepsilon}^e = C^e \dot{\sigma} \]  

where \( C^e \) is elastic compliance matrix which is determined by the first Hookean spring with elastic modulus \( E_0 \). The viscoelastic model described by the Kelvin element is written as:

\[ \dot{\varepsilon}^{ve} = C^{ve} i \sigma \]  

\[ C^{ve} = \begin{bmatrix} a & b & b & 0 & 0 & 0 \\ b & a & b & 0 & 0 & 0 \\ b & b & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\eta_d^{ve} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\eta_d^{ve} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/\eta_d^{ve} \end{bmatrix} \]  

\[ a = \frac{1}{9\eta_V^{ve}} + \frac{1}{3\eta_d^{ve}} \quad b = \frac{1}{9\eta_V^{ve}} - \frac{1}{6\eta_d^{ve}} \]
\[
\eta_v^w = \frac{1}{3(1-2\upsilon)} + \frac{\eta}{\exp\left(-\frac{E}{\eta} t\right)} \quad \eta_d^w = \frac{1}{2(1+\upsilon)} + \frac{\eta}{\exp\left(-\frac{E}{\eta} t\right)}
\]

(6-8d)

where $C^w$ is viscoelastic matrix which is determined by the Kelvin element.

### 6.5.3 Results of numerical simulation

The numerical modelling involves a large quantity of parameters which describe the retention capacity and water flow behaviour of soil. A series of sensitivity analyses was firstly carried out on some of these parameters using a pure hydraulic model to find the sensibility of the drawdown in PW, OB1 and OB2 to these parameters. Results show that the drawdown is sensitive to most of these parameters, especially the retention curve of waste and the intrinsic permeability of the pumping well and waste.

Next, these relatively sensitive parameters were estimated and adjusted by trial and error to achieve the best possible match to the time drawdown curves through the pure hydraulic modelling. It was found that the drawdown for the recovering stage cannot be agreeably reproduced even by adjusting skin layer property and the heterogeneity of waste. The leachate level in all wells recovers much slower and cannot recover to the pretest static level at the end of the recovering stage.

Another Code Visual Transin, which is especially designed for hydrology, was tried to simulate the same pumping test. It was found that agreeable time drawdown curves still cannot be obtained. The detailed results from Visual Transin can be found in the Appendix 6-A.

Only when including the mechanical behaviour of the waste, was a good fit to the field measurements achieved. The drawdown evolutions at three wells and at various pumping stages are quite well reproduced with parameters list in Table 6-3. The parameters for skin layer and waste are set to the same. Figure 6-10 gives the comparison between measured drawdowns and simulated results at the pumping well, OB1 and OB2. OB3 and OB4 are connected; therefore, they are excluded from the simulations. Drawdown curves for the pure hydraulic case with parameters in Table 6-3 are also presented in Figure 6-10. The anisotropic ratio of waste is found to be 10. Darcy conductivity of the waste is calculated to be $k_{darcy} = \rho_l g \left(\frac{k_{w}}{k_{r} l}\right) \mu \approx (4.5 \times 10^{-7} - 5.5 \times 10^{-6})$ m/s with an average value of $3 \times 10^{-6}$ m/s. The results show quite significant discrepancies between hydraulic modelling and coupled HM modelling. This implies the necessity of including the flow model in a deforming porous medium, and as a consequence variable hydraulic properties with deformation proceeding.
Although the fit to the drawdown-time curves during the recovering stage was significantly improved by considering viscosity of waste in the numerical simulation, the obvious delay of surface rebound in response to the leachate level recovery cannot be obtained. The comparison of the measured and simulated settlement rates at the distance of 2 m, 10 m and 20 m to the pumping well are shown in Figure 6-5. There is an immediate surface rebound when the pumping stops, while in reality the settlement continued several days before rebound was observed. The maximum surface settlement profile given by the numerical simulation at the 22\textsuperscript{nd} day is quite close to the surface settlement measured at the 25\textsuperscript{th} day as shown in Figure 6-6. The comparison indicates that the conventional mechanical model used in the numerical simulation is capable of predicting the ultimate magnitude of surface settlement, but not adequate to reproduce the time evolution of the settlement. The obvious delay of surface movement responding to leachate level variation is probably attributed to irreversible deformation of the waste (e.g. plasticity and/or biodegradation induced deformation) which cannot be simulated by viscoelastic model.

### Table 6-3: Principle parameters for waste in the numerical simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of solid phase, (kg/m\textsuperscript{3})</td>
<td>1800 (kg/m\textsuperscript{3})</td>
</tr>
<tr>
<td>Elastic modulus, $E_0$ (MPa)</td>
<td>50 (0-12 days)</td>
</tr>
<tr>
<td></td>
<td>9.5 (12-46 days)</td>
</tr>
<tr>
<td>Residual modulus $E$ (MPa)</td>
<td>0.35</td>
</tr>
<tr>
<td>Viscosity of waste matrix, $\eta$ (MPa.s)</td>
<td>$3 \times 10^8$</td>
</tr>
<tr>
<td>Poisson ratio, $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Horizontal intrinsic permeability (m\textsuperscript{2})</td>
<td>$5.5 \times 10^{-14}$</td>
</tr>
<tr>
<td>Vertical intrinsic permeability (m\textsuperscript{2})</td>
<td>$4.5 \times 10^{-15}$</td>
</tr>
<tr>
<td>Leachate viscosity, $\mu$ (MPa.s)</td>
<td>$1 \times 10^{-9}$</td>
</tr>
<tr>
<td>$P_0$ in the retention curve, (MPa)</td>
<td>0.035</td>
</tr>
<tr>
<td>Shape function of retention curve, $n$</td>
<td>0.015</td>
</tr>
<tr>
<td>Residual saturation, $S_{rl}$</td>
<td>0.1</td>
</tr>
<tr>
<td>Maximum saturation, $S_{ls}$</td>
<td>1.0</td>
</tr>
<tr>
<td>Initial porosity, $\phi$</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Figure 6- 10: Comparison of the drawdown-time curves of the numerical simulation and the field measurements.
6.6 Discussions and additional needs

Viscoelastic model is proved to be capable of describing the long-term settlement evolution in MSW landfills under monotonous loading (Yu et al., 2010). However, this kind of model does not reveal the real stress-strain mechanism of MSW. It is only a mathematical simplification used to capture the typical deformation curve characterized for MSW landfills. For most of the MSW landfills, the waste is constructed layer by layer and the load is exerted monotonously. Thus the application of viscoelastic model to describe waste deformation is successful. With respect to loading-unloading responses and other complex stress paths as what happens in a pumping test, mechanical constitutive models especially developed for MSW considering stress history, creep and biodegradation are indispensable. Including the advanced mechanical models in the numerical analysis of the pumping test, such as models by Sivakumar Babu et al. (2010) and McDougall (2008), is expected to be able to provide more agreeable results.

In the case of Coll Cardus pumping test, the set of parameters given in Table 6-3 which gives a good fit to the measured drawdown curves does not necessarily imply that these parameters are the real properties of the MSW material under investigation. In fact, this cannot be logically inferred because all inverse problems having multiple solutions (Chapuis et al., 2005). Other set of parameters based on different retentions curves and relative conductivity models could also be found to give a good match to the field data.

Experimental reveals that the hydraulic parameters vary significantly with the effective overburden stress (Beaven and Powrie, 1995). Furthermore, the suction-moisture retention curve is also expected to vary with effective stress state. The numerical modeling presented in this paper does not developed to accommodate such a variation. Before a suitable method of coupling between hydraulic properties and mechanical deformation being determined, the simplified hydraulic formulation is still chosen to be valuable.

6.7 Conclusions

Pumping test is a reliable method for getting insight into the hydraulic properties of MSW. Interpreting test results relies much on the comprehensive understanding of the coupled mechanical and hydraulic behaviour of MSW. This paper presents the results of a long-duration pumping test with variable pumping rate in a municipal solid waste landfill. Measuring surface displacement during pumping test provides a feasible method for evaluating mechanical behaviour of refuse. Measurements of the surface settlement near the pumping well reveal that six times more settlement was triggered when the pumping rate
increased from 0.2 L/s to 0.52 L/s after 12 days since pumping started. Delayed rebound of landfill surface was observed six days after the pumping stopped. These phenomena indicate the possible creep and/or biodegradation-induced deformation of waste.

An estimation of hydraulic properties was obtained through numerical analysis of the leachate pumping test, which considers the water release due to the mechanical deformation, the water retention capacity of waste and variations of the relative permeability with water saturation. The hydraulic conductivity determined in this study varies between $4.5 \times 10^{-7} - 5.5 \times 10^{-6}$ m/s with anisotropic ratio of 10. These values are quite similar to previous reported data from pumping tests at MSW landfills, normally from $10^{-6}$ to $10^{-5}$ m/s. The HM coupled simulations based on theories established for describing liquid flow in saturated and unsaturated porous media are proved to be applicable to leachate migration in MSW landfills.

The numerical analyses indicate that the viscoelastic model used in the numerical simulation is capable of predicting the ultimate magnitude of surface settlement, but not adequate to reproduce the time evolution of the settlement. The obvious delay of surface movement responding to leachate level variation is probably attributed to plasticity and/or biodegradation induced deformation.
Appendix 6-A: Results of hydraulic analyses
Appendix 6-B: Photos of the pumping test

Figure 6-B-1: Construction of the well.

Figure 6-C-2: Installation of the well.
Figure 6-C-3: Pumping well and observation wells.

Figure 6-C-4: Photos of the settlement measuring plates.
Chapter 6 - Numerical analyses of a leachate pumping test at a MSW landfill

Figure 6-C-5: Pumping well.
CHAPTER 7

Conclusions and recommendations

7.1 Summary and conclusions

This thesis investigated the settlement mechanism for MSW and clarified the relationship between mechanically and biologically induced settlement. The process of degradation and the subsequent collapse of waste solids are time-dependent and implicitly incorporated into a K-H rheological model. This mechanical model was then coupled with LFG generation and transport model and applied to various aspects related to post-closure behaviours of the MSW landfill, including prediction of long-term settlement, dynamic evaluation of the slope stability of waste fills and gas emission control. The parameters required in the model were obtained for conventional MSW landfills which will help the engineers in the landfill design and operation. The main findings of this work are summarized below:

1. Appropriate prediction of waste unit weight profiles is a first step for any other engineering analysis of MSW landfill performance. This thesis has provided a model to simulate the temporal and spatial evolution of unit weight through calculating time variations of strain with respect to depth within MSW landfill. The total strain includes mechanical strain caused by the weight of subsequent lifts during filling, and time-dependent strain due to both creep and degradation of waste during and post filling. Time variations of the waste unit weight at any depth within a MSW landfill depend on the competition between two contrary effects: decrease due to degradation and increase due to mechanical deformation of the solid matrix. Solutions of the unit weight within the MSW landfill were expressed analytically in the Laplace transform domain.
Typical parameter values were obtained for three representative MSW landfills with low, typical and high near-surface unit weight. The good agreement between model simulations and in situ data using conventional parameters suggests that the proposed model is applicable in practice. Although compressive strain within the waste column keeps increasing with depth and time, there is no single variation trend of waste unit weight along the whole depth of the landfill. Unit weight decreases with time in the upper portion of the landfill because degradation overrides mechanical deformation. The opposite occurs in the lower portion of the waste.

2. Analytical solutions have been developed to predict one-dimensional settlement and gas flow for both single-layered and multi-layered MSW landfills. Mechanical compression of the solid skeleton is coupled with gas pressure using K-H viscoelastic model. The comparison with field records at two landfill sites proves that the proposed model can well reproduce the time evolution of settlement and predict gas flux in horizontal LFG collection systems. Parameter values for prediction of long-term settlement and gas flow have been obtained for conventional MSW landfills.

A sensitivity analysis showed that the rate of settlement is not greatly influenced by gas conductivity, \( k_a \), and cover property, \( L_c \), although excess gas pressure is sensitive to these parameters. In the typical MSW landfill, excess gas pressure is of orders less than gravitational stress and the coupling effect between gas pressure and mechanical compression is too little to be noticeable. The coupling effect may become apparent under the cases of low gas conductivity, deep landfills, bioreactor landfills, or inefficient gas extraction system. Under these circumstances, the increased gas pressure may apparently delay the settlement of the landfill.

3. An analytical solution to predict gas flow to a vertical gas extraction well has been presented in this study. The solution accounts for the effects of exponential decaying gas generation, mass storage variations due to compression of gas and refuse, dissolution of gas components and porosity enlargement due to solids degradation. Mechanical compression of the refuse is coupled with gas pressure using K-H rheological model. The solution was obtained analytically in Laplace transform domain for excess gas pressures. The solution is applicable to an anisotropic landfill with a low-permeable final cover, and bounded at some depth by either the water table or an impermeable liner. Storage terms are only relevant during early time (i.e., up to a few days). Afterwards, gas flow pattern is quasi-steady state solely controlled by gas generation rate. Therefore, the model proposed in this paper can be simplified to be steady-state flow with fixed gas generation rate. This implies that optimum extraction strategy can be obtained with little attention for past extraction. This facilitates real time optimal operation and control.
Parametric studies indicate that the solution to the proposed model is convenient in the optimization of gas extraction systems, including well distance, cover properties and vacuum imposed in the extraction well. The expression of gas fluxes in the well, $Q_{\text{well}}$, shows that there exists a quasi-linear relationship between vacuum imposed and $Q_{\text{well}}$. The study shows that both the vacuum applied in the well and anisotropic ratio of refuse will clearly improve the well efficiency in recovering biogas. Radius of influence increases from 0.5 H to 2 H as well pressure varies from zero to -4.5 kPa, and increases from 0.92 H to 2.6 H as the permeability anisotropic ratio varies from 1 to 10. The phenomenon that more atmospheric air enters into the landfill in the vicinity of the active extraction well through the top cover indicates that increasing the vacuum imposed in the extraction well will not result larger radius of influence proportionally. An economical balance should be carefully sought among the vacuum imposed, well distance and properties of the top.

The proposed model is a promising engineering tool for preliminary approximations and guidance in the design of LFG control systems. Despite the fact that the model is established based on some simplifications, it is capable of describing quite accurate settlement evolutions, and gas flow patterns within MSW landfills.

4. A hybrid method for quasi-three-dimensional slope stability analysis was developed in this study for MSW landfills. This method computes an 'equivalent' three-dimensional factor of safety (FoS) from the results of two-dimensional analyses for a series of evenly spaced cross sections within the potential sliding body, aligned with the direction of sliding. The two-dimensional analysis at each cross section was realized by combining finite element stress analysis with the concepts of limit equilibrium method. In the finite element stress analysis, time-dependent deformation of MSW based on viscoelastic model (K-H rheological model) was used to reflect the effect of the process of biodegradation.

The model was applied to a MSW landfill in Spain. This landfill lies over strong metrophos rocks. Therefore, deep slip surfaces could be neglected. Two-dimensional slope stability analysis gives a minimum FoS of 1.2 for the potential slip surfaces located in the front slope, about 40 m deep within the landfill. The potential three-dimensional slide mass is located with a dimension of approximately 230 m in length, 108 m in width and 100 m in height, with a FoS of 1.8. FoS from three-dimensional analysis is 50% higher than that from two-dimensional analysis because the confining effect of the valley-shaped geometry of the landfill as well as the resistances at two lateral surfaces of the slide mass were taken into consideration.

Results show that the hybrid method for quasi-three-dimensional slope stability analysis based on FEM results is easy to perform and can serve as an engineering
tool in the preliminary estimate of the FoS as well as the approximate position and extent of the potential slide mass.

Influences of time elapsed after landfill closure, leachate level as well as unit weight of waste on FoS were investigated. The surface of the landfill was dynamically monitored by continuous updating the element coordination in the finite element analysis. Results show that factor of safety does not vary much with time, varying between 1.2-1.31. One reason is because two key parameters, unit weight and shear strength, are determined by effective stress and not considered to vary with time in this case study. The other is the shape of the front slope does not flatten much after closure because the waste at that area was deposited in the early 80’s and most of the biodegradation settlement has already finished by the time of closure. FoS is found to increase rapidly with lowering the leachate level by the first 20 meters. Afterwards, lowering leachate level cannot increase the FoS anymore because a newly-formed shallow slip surface comes up in the upper landfill, which is located above the leachate level and will no longer be affected by the position of the leachate level. In this study, strength parameters are assumed increasing linearly with the effective vertical stress which is determined directly by the unit weight of waste. Therefore, increasing the initial density of waste increases monotonously the slope stability.

5. Pumping test is a reliable method for getting insight into the hydraulic properties of MSW. A long-duration pumping test with variable pumping rate in a municipal solid waste landfill was performed in this study. Interpreting test results relies much on the comprehensive understanding of the coupled mechanical and hydraulic behaviour of MSW. Measurements of the surface settlement near the pumping well reveal that six times more settlement was triggered when the pumping rate increased from 0.2 L/s to 0.52 L/s after 12 days since pumping started. Delayed rebound of landfill surface was observed six days after the pumping stopped. These phenomena indicate the possible creep and/or biodegradation-induced deformation of waste.

Estimations of hydraulic properties was obtained through numerical analysis of the leachate pumping test, which considers the water release due to the mechanical deformation, the water retention capacity of waste and variations of the relative permeability with water saturation. The hydraulic conductivity determined in this study varies between $4.5 \times 10^{-7}$ - $5.5 \times 10^{-6}$ m/s with an anisotropic ratio of 10. These values are quite similar to previous reported data from pumping tests at MSW landfills, normally from $10^{-6}$ to $10^{-5}$ m/s. The hydro-mechanical coupled simulations based on theories established for describing liquid flow in saturated and unsaturated porous media are proved to be applicable to the leachate migration in MSW landfills.
The numerical analyses indicate that the conventional viscoelastic model used in the numerical simulation is capable of predicting the ultimate magnitude of surface settlement, but not adequate to reproduce the time evolution of the settlement. The obvious delay of surface movement responding to leachate level variation is probably attributed to plasticity and/or biodegradation induced deformation.

7.2 Recommendations for the future research

The coupled model presented in this paper focuses only on solid phase and gas phase interactions. The water saturation is assumed to be a constant which facilities the derivation of analytical solutions. Therefore, it is restricted to relatively dry or fully drained landfills. It would be very interesting to test if this model can be extended to include liquid phase and consider the interaction among solid, gas and liquid phases in the future.

For simplicity, the mechanical-gas coupled model presented in this study only considers a single gas component. No opportunity is given to represent characteristics of various LFG components and their interactions. Multi-component gas generation and transport model needs to be considered in the future. In fact, degradation is simulated by simple experimental formulas that result from first order kinetics. In reality, the process is much more complex. Full simulation of the degradation process is also a possible line of future research.

Delayed mechanical responses to pumping rate variations need to be simulated using appropriate mechanical models especially designed for MSW.
References


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